1. **Prove or disprove.** For any Hex position $H$ where neither player has yet won and White has a 2nd-player winning strategy $S_2$, White also has a 1st-player winning strategy $S_1$. Hint: strategy stealing.

2. **Claim.** For any game $G$, $G + 0 = G$. Complete the following **proof by induction.**

Recall that in the definition $G + H = \{G^L + H, G + H^L \mid G^R + H, G + H^R\}$

$G^L$ is the set of all games that are left options of $G$,

$G^L + H$ is _______________________________________________________________________

and the comma means “take the union of these two sets”.

$0$ is the game with no options for R or L, i.e. $0 = \{\mid \}$. Let $G = \{G^L \mid G^R\}$ where $G^L$ is defined as

____________________________________________________________________________________

By the definition of game sum, $G + 0 = \{G^L + 0, G + 0^L \mid G^R + 0, G + 0^R\}$. (1)

But $G^L + 0 = G^L$ because _______________________________________________________________________.

Similarly, $G^R + 0 = G^R$.

Also, the set of games $0^L$ is empty because _______________________________________________________________________,

so the set of games $G + 0^L$ is empty because here the + operator means

____________________________________________________________________________________.

Similarly, the set of games $G + 0^R$ is empty.

So the set $G^L + 0, G + 0^L$ equals the set $G^L, \phi$ which equals the set $G^L$.

Similarly, $G^R + 0, G + 0^R$ equals the set $G^R$.

So, from (1) we have $G + 0 = \{G^L \mid G^R\}$ which is equal to ______________________________________________________________________.

3. 0, 1, −1, * are (resp.) games $\{|\}$, $\{0\}$, $\{|0\}$, $\{0|0\}$. $Q$ is $\{1|−1\}$. Express each using only symbols $\{\} |$.

4. (i) Let $Z$ be any game in outcome class $P$. Let $G$ be any game. Explain why the outcome class of $G$ equals the outcome class of $G + Z$. (ii) Using (i) and the definition of game equality, prove that $Z = 0$.

5. (i) Find domineering games $A, B$ in respective outcome classes $L, N$ such that $A + B$ is in $L$.

(ii) Repeat (i), so that $A + B$ is in $N$. (iii) Prove that there is no such $A, B$ with $A + B$ in $R$ or $P$.

6. For the 5x6 chomp position with the upper-right cell removed (so, 29 cells remain), find all winning moves. Hint: use the program from class.

7. (20,21) is the chomp position with 2 rows: the top row has 20 cells, the bottom has 21. Prove that this is a P-position. Hint: pair cells so that if the opponent plays in one, you play in the other.