In domineering, L’s dominoes are vertical (each looks like an l), R’s are horizontal. In this course, unless stated otherwise, game means combinatorial game.

1. For (i) domineering (ii) hackenbush (iii), give two P-positions, two N-positions, two L-positions, two R-positions or explain why this is not possible.

2. Give the outcome class for the hackenbush game in the video

https://www.youtube.com/watch?reload=9&reload=9&v=DrtMWZbh1so (N, P, L, R)

3. Consider a game G where R plays first. In your own words, prove by induction: R wins or L wins.

4. A game tree shows all possible continuations of a game, for any (not necessarily alternating) order of play. We can draw a game tree G like this:

- each node is a position,
- the root is G’s position,
- each subtree is a subgame (i.e. an option of the associated game),
- each left child of a node (draw with a solid branch) is a left option of that node’s position,
- each right child of a node (draw with a dashed branch) is a right option of that node’s position.

Pruning isomorphic options leaves the reduced game tree. For each position (i,ii,iii,iv), draw the reduced tree, describe it in game theory notation, e.g. $G = \{G^L|G^R\}$, and if the game has a name, give its name.

(i) 1×1 board (ii) 1×2 board (iii) 2×1 board (iv) 2×2 board.

Hints. For (i), the tree has a root node with no children, $G = \{ \mid \}$, and its name is 0 (zero). For (iv) the tree has 5 nodes: root A has only one left child B, the 2×1 position. B has no right child, etc.

5. Draw the reduced game tree for 2-rows-3-columns domineering.

6. Give the number of nodes in the reduced 3×3 domineering tree.

7. (i) Explain why the usual rules for chomp do not satisfy the requirement of being a combinatorial game. (ii) Explain how to modify the rules so that the game is the same (the same player who wins with the usual rules wins with the modified rules) but the modified game is a combinatorial game.

8. Give a strategy-stealing proof that the first player wins chomp on the $m \times n$ board with $m \geq 2$. 