1. (i) Find a 3-pile nim position with at least 3 winning moves, or prove that no such position exists.

(ii) Find a 4-pile nim position with at least 4 winning moves, or prove that no such position exists.

2. There are $3^4 = 81$ different ways to assign black, white, or empty to the cells of a $2 \times 2$ game board. How many of these are legal Go positions? Hint: count illegal positions and subtract from 81. E.g. four black stones is illegal: the black group has no liberties.

3. In Go, can a legal non-pass move leave the empty board? Explain briefly.

4. Below is a diagram with some legal $2 \times 2$ Go positions. Draw a line between two positions if there is a legal move from one position to the other.

5. Assume that you complete the diagram above by adding the remaining legal Go positions, and all necessary lines. The resulting diagram is called the $2 \times 2$ Go transition graph. Explain why the number of paths in this diagram that start from the empty position is the same as the number of leaves in the $2 \times 2$ Go game tree (which accounts for every legal $2 \times 2$ Go game).

7. If you solve 2×2 Go using minimax (with no cutoffs) roughly how many leaf nodes will be in the search tree? What if you use alpha-beta search, with the pass move always considered as a last option? What if you use alpha-beta search, with the pass move always considered as a first option? (See http://tromp.github.io/java/go/twoxtwo.html for answers.)

8. (i) For a Go position, explain what it means to say that an empty point reaches no Black stones. (ii) In your own words, define the rules of Go. (iii) For a Go position, Black’s net score is \(bs + bt - (ws + wt)\), where \(bs, bt, ws, wt\) are respectively the number of black stones, black territory points (reach only black stones), white stones, white territory points. For 2×2 Go, give all possible integers that can be the net score of a position. (iv) Repeat (iii) for 3×3 Go.

9. Read http://webdocs.cs.ualberta.ca/~hayward/396/ssgo.pdf up to and including section 2×2 Go. A proof tree is a particular strategy — in tree form — whose minimax value establishes a bound on the overall minimax value.

(i) For the 1×5 Go state with White to play after Black has played at location 1 (leftmost cell), give a (White) proof tree showing that Black’s minimax net score is at most −5. Explain why this implies that, for this state, Black’s minimax net score is exactly −5.

(ii) For 2×2 Go, the above article has a proof tree showing Black’s minimax net score is at least 1. Draw a (White) proof tree showing that Black’s 2×2 Go net minimax score is at most 1. (I did this in class.)

10. Give the rules of (i) cram (ii) domineering (iii) Hackenbush.