In this course, unless otherwise stated, for Go rules we use Tromp-Taylor, no suicide, positional superko.

1. Using the nim formula, find all winning initial moves for each position. Show your work.

\[(15,13,11,6,2) \quad (3,5,7) \quad (1,3,7,15,31) \quad (31,31,31)\]

2. Prove the correctness of the nim formula.

3. Give a 4-pile nim position with exactly 3 winning moves. Explain briefly.

4. For \(a \geq 1\), prove by induction that the first player loses nim\((a, a)\). Do not use the nim formula.

5. For Go, explain (i) the two kinds of legal move (ii) the game-termination condition (iii) how the game is scored.

6. In Hex, a state is a position and the player-to-move. In Go, a state is a position and a move history. Explain why move history is needed in a Go state but not a Hex state.

7. The usual terminating condition in a combinatorial game is that the game ends when the player-to-move has no legal moves, and that player loses. A combinatorial game is pure if it has the usual terminating condition and also satisfies these properties: 2-player, alternate turn, deterministic, finite. For each of Go, Hex, nim, is this a pure combinatorial game? Explain briefly.

8. Answer these questions for the position below, from Figure 3.1 in Mathematical Go: Chilling Gets the Last Point by Berlekamp and Wolfe. (i) Give the number of black groups, white groups, black stones, white stones, black territory, white territory and (assuming komi 6.5) the final score. (ii) Assume White now makes a non-pass move, and then both players pass. What is White’s best move? Explain briefly. (iii) Repeat (ii) for Black. (iv) From the position, assume White makes some number non-pass moves and that Black passes after each and then White passes. Assuming komi 0, what is the best score that White can achieve? Explain briefly. (v) Repeat (iv) with colors exchanged (Black makes non-pass moves).