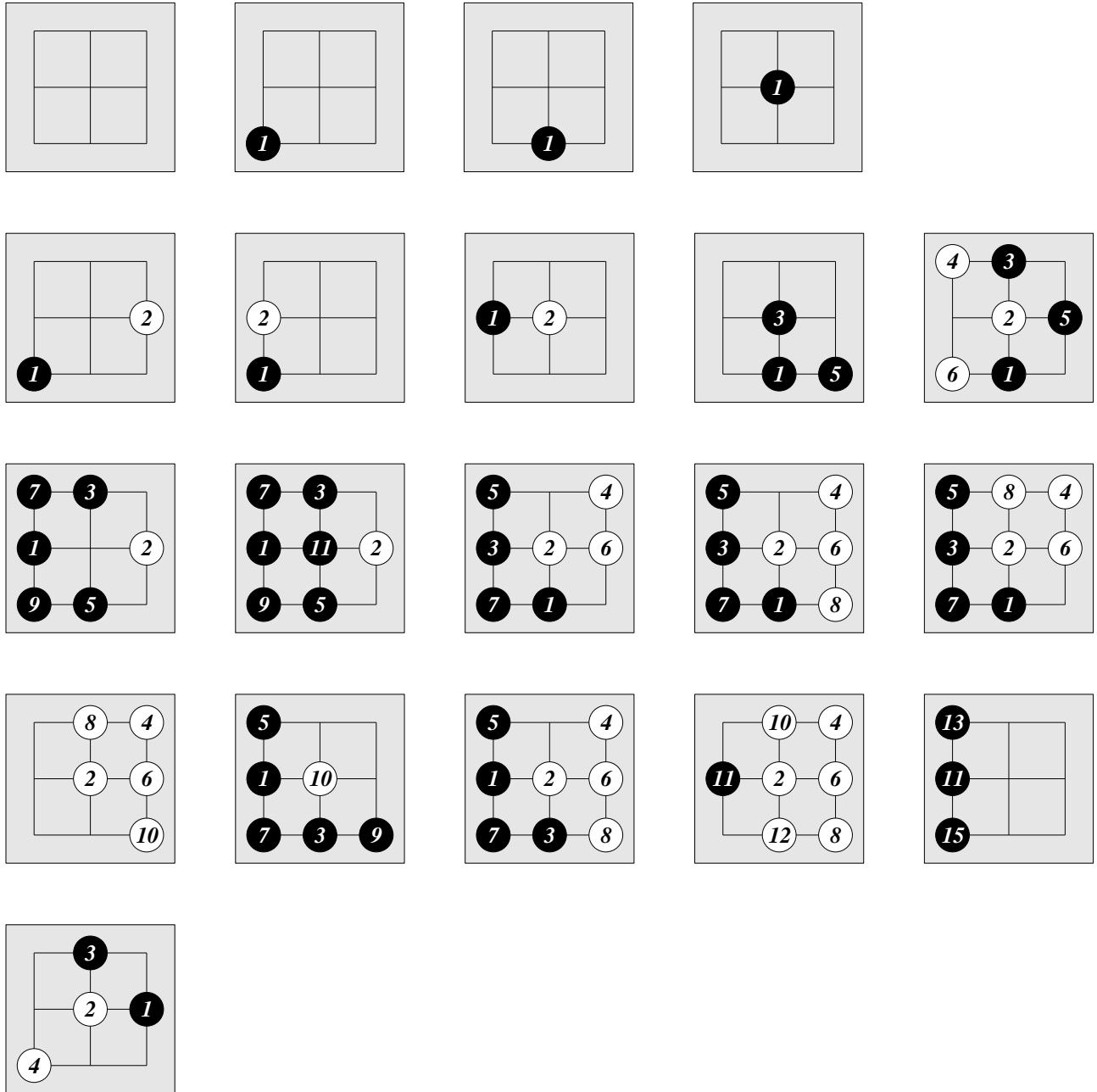


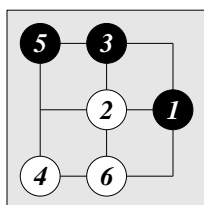
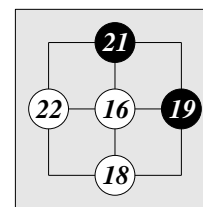
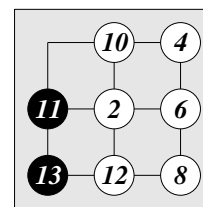
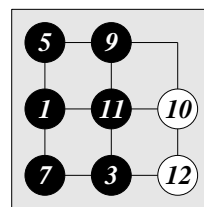
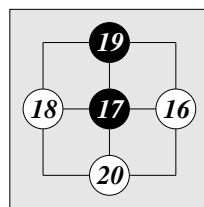
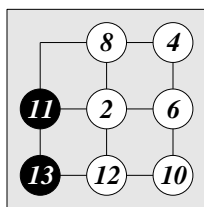
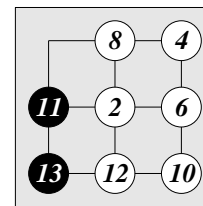
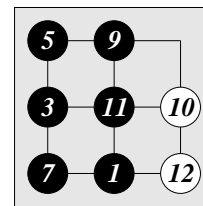
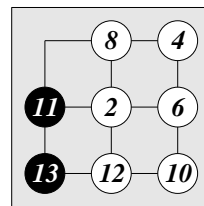
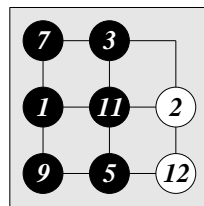
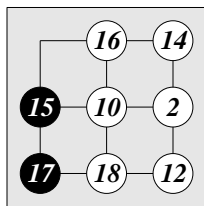
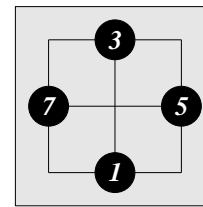
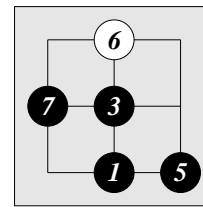
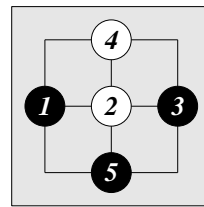
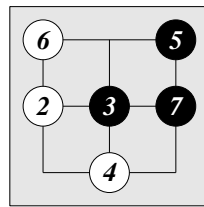
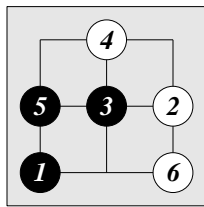
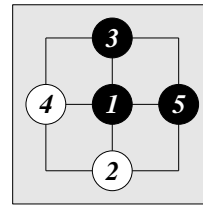
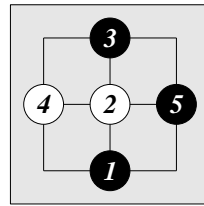
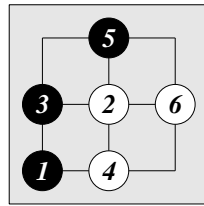
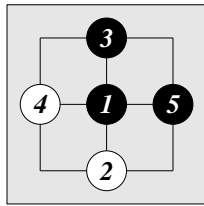
3x3 go puzzles

Rules: Tromp-Taylor, no-suicide, positional superko. For each state, give minimax score and a principal variation.



solutions

+9 -9 +3 +9
 -1 -1 +3 +9 +9
 -4 +4 -4 +4 -4
 -4 -9 +4 -4 -9



One solution, in detail

Below left is a 3x3 Go puzzle: what is Black's best move 3? And final minimax score? Solution. B has three reasonable options (the others are self-atari that allow W to kill all): pass, a2 or (isomorphic) b1. Each is a best move: minimax score W wins by 1. Below right is one final position.



Here is a proof that these are the only best moves for B.

On move 3, B has 8 possible moves, including pass. But if move is anything other than 3.B[b2], then 4.W[b2] and (since B's 3rd move occupies at most one cells) at least two of a2,b3,b1 are empty, so after 5.B[?] at least one of these three is still empty, so W can play one of them on move 6, forming either a bent-3 or an inline-3, guaranteeing that W will kill all B stones, so final minimax score will be B-9.

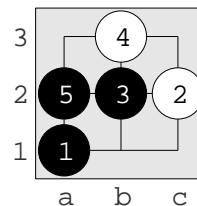
So 3.B[b2] is forced, otherwise W kills all. And what happens next? 4.W[b3] is also forced. Otherwise, 5.B[b3] and on move 7 B forms either a bent-3 with 7.B[a2] or 7.B[b1], and B kills all.



For move 5 the countermoves vary. There are four cases to consider: B has five legal moves but two are isomorphic.

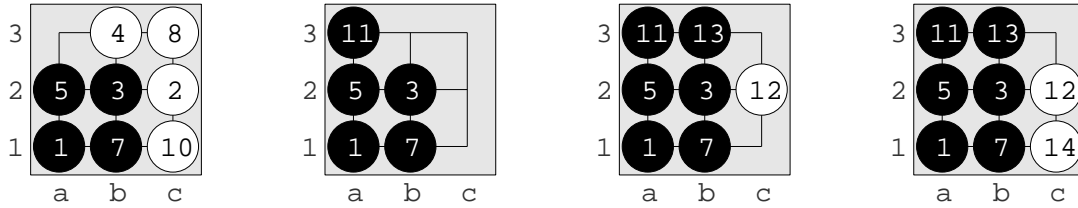
Case 0. If 5.B[pass] then W can pass, so final score W +1. (This is a best move for W. Another best move is to play 6.W[c1], then 7.B[a2], and from here the minimax score is W wins by 1. The proof is similar to the proof in Case 1 below.)

Case 1. If 5.B[a2] then W has 5 possible replies. 5.B[a3] and 5.B[c1] both allow W to kill all. 5.B[a2] and 5.B[b1] are isomorphic. So there are only two reasonable non-isomorphic B moves, they are both best moves for B.



Case 1-0. 6.W[pass]. B can reply 7.B[b1]. Now if W attacks (either 8.W[a3] or 8.W[c1]) then B can kill all; if W passes then B can pass, B wins by 1; if W fills its own eye (8.W[c3]) then B can pass and W must pass (otherwise B can win by at least 1, as explained next), B wins by 1. In each case B wins by at least 1.

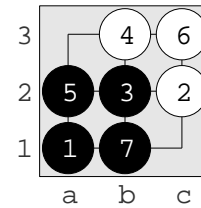
So what happens in case 1-0 when W fills its own eye with 8.W[c3], B passes, and W attacks with 10.W[c1] or (isomorphic) 10.W[a3]? Answer on the next page?



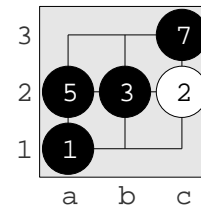
Assume 10.W[c1]. Then 11.B[a3], 12.W[c2] is forced (otherwise 13.B[c2] forms a bent-3 and B can kill all), 13.B[b3] and B passes the rest of the game (unless W plays all three of c1,c2,c3, capturing all B ... but then B kills all!). So the game ends as in one of the two above rightmost positions, B wins by 4 or 5. This concludes case 1-0: in each subcase, B wins by at least 1.



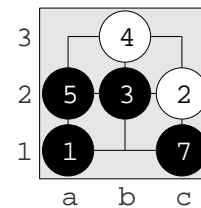
Case 1-1. 6.W[c3], 7.B[b1]. This is similar to case 1-0: B wins by at least 1.



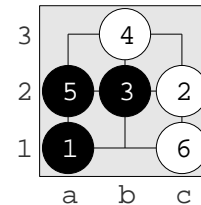
Case 1-2. 6.W[a3], 7.B[c3] and B can kill all.



Case 1-3. 6.W[b1], 7.B[c1] and B can kill all.



Case 1-4. 6.W[c1], 7.B[pass]. If W attacks with either 8.W[a3] or 8.W[b1] then B captures and kills all. If W passes the game ends, W wins by 1. Finally, what if W fills its own eye with 8.W[c3]? As in Case 1-1, B takes b1 with 9.B[b1], and B wins by at least 1.



Case 1 conclusion: after 5.B[a2] W's only best response is 6.W[c1], W wins by 1.