

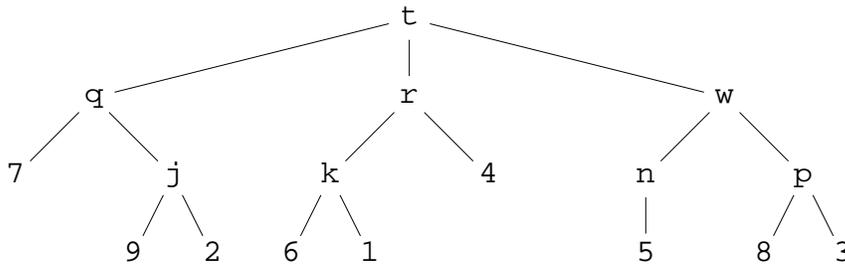
## 2026 cmpu 670 homework 4

Explain each answer carefully. For a game tree with leaf nodes labelled with scores, the (1st-player) *minimax value* of each node of the tree is defined as follows: for leaf nodes, the score; for 1st-player-to-move nodes: the max, over all children, of the child nodes' values; for 2nd-player-to-move nodes: the min, over all children, of the child nodes' values.

1. For any game and player  $X$  and  $x$ -strategy  $S$ , define the minimax score of  $S$ .

**answer** It's the above definition with game tree replaced with the player's strategy tree.

2. For this game tree, give the minimax value for each non-terminal node.



**answer** j 9 q 7 k 6 r 4 n 5 p 8 w 5 t 7

3. For a finished game of go, define a player's *net score* as their score (stones plus empty cells that reach only their color) minus their opponent's score. E.g. for  $2 \times 2$  go, empty board net score is 0, board with one black stone is B net score 4, W net score  $-4$ .
  - i) For the diagram on the next page, label each leaf node with Black's net score.
  - ii) What does the diagram show for  $2 \times 2$  go? Pick one answer.
    - a) first-player minimax net score is at least 1
    - b) first-player minimax net score is at most 1
    - c) first-player minimax net score is at least  $-1$
    - d) first-player minimax net score is at most  $-1$
    - e) first-player minimax net score is exactly 1
    - f) first-player minimax net score is exactly  $-1$
    - g) none of the above.



4. For  $2 \times 2$  go, give a diagram that shows one of the following:

- a) the second-player minimax net score is at least 1
- b) the second-player minimax net score is at most 1
- c) the second-player minimax net score is at least  $-1$
- d) the second-player minimax net score is at most  $-1$ .

5. What follows from the previous two questions:

- a) for  $2 \times 2$  go, the first player minimax net score is 1
- b) for  $2 \times 2$  go, the first player minimax net score is 0
- c) for  $2 \times 2$  go, the first player minimax net score is  $-1$
- d) none of the above.

6. Consider Tromp-Taylor no-suicide  $2 \times 3$  go. a) For each state below, assume that the state occurs in a game. Find the state's minimax score (score achieved by minimax play from that point on, i.e. each following move is minimax-best).

. . o \* to move  
. . . score \* 6

. . o \* to move  
. . o score \* 6

\* \* . o to move  
\* \* . score \* 6

? \* ? (one or more ? empty)  
? \* ? o to move, score: \* by 6

\* . ? (one or more ? empty)  
. \* ? o to move, score: \* by 6

\* \* . o to move  
\* . . score: \* 6

b) (ignore for now) Prove that minimax score is 0.

c) (ignore for now) Prove that the only minimax first move is pass.

7. Explain how to prove directly (without using any theorems) that the impartial game  $g$  with move options  $\{ *0, *1, *2, *4, *7 \}$  equals the game  $*3$ .
8. Prove that a game  $k$  with move options  $\{ *k_1, *k_2, *k_3 \}$  is equal to  $*\text{mex}(k_1, k_2, k_3)$ . This question is a special case of what theorem?
9. Prove by induction that the  $m \times n$  chop position equals  $*(m - 1) + *(n - 1)$ .
10. a) Give the canonical form of the game  $g = \text{chop}(3 \times 4) + \text{bricks}(5) + \text{nim}(5)$ .  
b) If you play first on  $g$ , what move do you make?
11. Every finite normal-play game can be represented by a Konane position. Give the canonical form of each of these positions.

x o .	. o x . x .	x . x o .	. o x .
o x .	o . . . .	. . . . o	o . . x
. . .	. . . . .	. . . . .	. . . .