

## cmput 670 2026 homework 2

Assume  $A, B$  are normal-play games. For any game  $g$ ,  $t(g)$  is the game tree of  $g$ .

1. Recall these definitions:

- $A \equiv B$  if  $A + -B$  is in  $\mathcal{P}$ ,
- $A < B$  if  $-A + B$  is in  $\mathcal{L}$ , and
- $A \leq B$  if  $A \equiv B$  or  $A < B$ .

a) Prove that  $\equiv$  is an equivalence relation.

b) Prove that  $<$  is not an equivalence relation.

c) Prove that  $A \leq B$  if and only if Left has a 2nd-player-win strategy on  $-A + B$ .

2. a) For a normal-play game  $g$ , given  $t(g)$ , explain how to draw  $t(-g)$ .

b) Assume that game  $g$  is fuzzy. Using a), prove that  $-g$  is fuzzy.

c) Using b), prove that  $A||B$  if and only if  $B||A$ .

3. Prove that  $* \leq 1$ . Prove that  $*||0$ .

4. Call a normal-play game  $g$  *minimal* if every game  $h$  with  $t(h)$  a proper subtree of  $t(g)$  is non-equivalent to  $g$ .

a) Prove that  $\{1, -1|1, *\}$  is not minimal.

b) Explain why there are exactly 4 minimal games with depth at most 1.

c) Explain why there are at most  $2^4 \times 2^4$  minimal games with depth 2.

d) Explain why there are at most  $6 \times 6$  minimal games with depth 2.

e) Explain why we call the game  $\{0|1\}$  one-half.

f) For each of these games, explain why Left's option to  $*$  can be reversed, and give the reduced game after Right reverses the move.

$$\{*\}$$

$$\{0, *\}$$

$$\{0, *|*\}$$

$$\{0, *|1\}$$

g) For each of these games, explain why Left's option to  $*$  cannot be reversed.

$$\{0, *| - 1\}$$

$$\{0, *|0, *\}$$