

first name

last name

id#

(2 + 3) + (2 + 3) + (3 + 2) marks

50 min

closed book

no devices

3 pages

page 1

1. Find the canonical form of the 4-pile nim game $\text{nim}(13, 27, 14, 19)$. **Show your work.**

Hint: 1 1 0 1, 1 1 0 1 1, 1 1 1 0, 1 0 0 1 1.

Which theorems if any are you using in your answer?

2. Prove directly (without using any theorems) that the impartial game g with move options $\{ *0, *1, *4, *7 \}$ equals the game $*2$.

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page 2

3. a) Prove that the 2×3 chop position equals $*3$.

b) For each $n \geq 1$, prove that the $n \times n$ chop position equals $*0$.

(You can give a strategy: you don't need to argue by induction.)

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page 3

4. a) Give the canonical form of the game $g = \text{chop}(2 \times 3) + \text{bricks}(5) + \text{nim}(3)$. **Show your work.**

b) If you play first on g , what move do you make? **Justify carefully.**