first name **last name** **id#**

$(2 + 3) + (2 + 3) + (3 + 2)$ **marks** **50 min** **closed book** **no devices** **3 pages**

Recall: ptm is player-to-move, optm is opponent of ptm. A *state* (X, P) is a game position X together with ptm P. Recall Zermelo's theorem: for any finite-move alternate-turn 2-player game in which each terminal state (T, P) is ptm-win and optm-lose, ptm-lose and optm-win, or ptm-draw and optm-draw, ptm has a winning strategy (in which case we call (T, P) a W-game), or optm has a winning strategy (an L-game), or each player has a strategy that wins or draws against any opponent strategy (a D-game).

Consider a W/L/D game with players P and Q and a state (S, P) from which P has five possible moves, yielding states $(Y_1, Q), \ldots, (Y_5, Q)$. Assume that Z's theorem holds for these five states. Also assume each (Y_j, Q) except for $j = 2$ is an L-game and that (Y_2, Q) is a D-game.

1. [5 marks] Prove/disprove: Z's theorem holds for (S, P).

2. [3 marks] For the 5-pile nim position below (binary numbers), find all winning moves. Show your work.

```
pile 1   1 0 1 0 1 1 0 1
pile 2     1 1 1 1 0 1 1
pile 3       1 0 1 1 0 1
pile 4   1 0 0 0 1 0 1 0
pile 5     1 1 0 0 1 0 0
```

3. [2 marks] Prove that for $n \times n$ Hex there exists a winning strategy for the first player.

4. [5 marks] Complete this drawing of a 2x2 go second-player (white) strategy that shows that white can (win or draw or) lose by at most 1. (Symmetric children of the root have been pruned.)