cmput 497/670 2024 homework 5

Explain all answers carefully. Questions from Devos/Kent.

1. Rose and Colin play each of these games. Give their minimax strategies. How can you answer this question without using an LP solver?

a)
```
5 4 3
6 2 0
```

b)
```
0 6 1
2 5 0
5 4 3
```

c)
```
5  6 4 7
1 11 3 0
7  9 5 6
```

2. Colonel Blotto and Count Baloney are opposing commanders in a battle. There are three locations valued 1, 2, 3, and each comman- der has two military units to deploy (both to the same location or split to different locations). If one commander sends more units to a location than the other, that commander captures the location and wins its value. If both commanders send the same number of units to a location, it is not captured and nothing is won at this location.

   (a) Model this conflict as a zero-sum matrix game.

   (b) Use the method of eliminating dominated (not necessarily

   strictly dominated!) strategies to reduce this to a 1×1 game.

   (c) What strategy would you recommend to these commanders and why?

3. A spinner randomly selects from $\{A, B, C\}$ choosing each with probability 1/3. Assume the spinner is spun three times, giving outcomes such as ABC or BBA.

   (a) How many possible outcome sequences are there? What is the probability each is selected?

   (b) Let f be the random variable denoting the total number of times the spinner selects A. For each integer $0 \leq j \leq 3$, determine the probability that f has value j.

   (c) What is the expected value of f?

4. For this 0-sum matrix game and mixed strategies $p = (2/3, 1/3)$ for Rose, $q = (1/5, 2/5, 2/5)^T$ for Colin, find the expected payoff for each strategy:

```
3  4  2
5 -2  7
```

   a) Rose row 2, Colin column 2

   b) Rose row 1, Colin q

   c) Rose p, Colin column 3

   d) Rose p, Colin q

5. Two-Finger Morra rules: simultaneously, each player shows 1 or 2 fingers with their left hand and guesses (how many fingers the opponent will show) 1 or 2 fingers with their right hand. If both guess correctly or both guess incorrectly, the game is a draw. If exactly one player guesses correctly, they win the sum of the number of fingers shown. a) Complete the payoff matrix below:

	show 1 guess 1	s1 g2	s2 g1	s2 g2
s1 g1	0	2	-3	0
s1 g2	-2			
s2 g1				
s2 g2				

b) Prove that the mixed strategy $(5/12, 7/12)$ `(s2g1, s1g2)` has minimax guarantee 0 for each player.

c) Find the Von Neumann equilibrium for this game.

6. For this 0-sum matrix game, Rose plays mixed strategy `(row 1, row 2)` with probabilities $(1/4, 3/4)$ and Colin plays mixed strategy `(row 1, row 2)` with probabilities $(3/4, 1/4)$.

2	-1
1	2

a) What is Rose's guaranteed payoff, against any possible strategy by Colin.

b) Give a Colin strategy that maximizes Rose's payoff (worst possible for Colin).

c) What is Colin's guaranteed payoff, against any possible strategy by Rose.

d) Give a Rose strategy that minimizes Colin's payoff (worst possible for Rose).

e) What is the Von Neumann equilibrium value of this game?

7. For this 0-sum matrix game, assume Rose plays mixed strategy `(row 1, row 2)` with probabilities (x, y).

2	3
4	1

a) give Rose's expected payoff against each of Colin's two pure strategies `(col 1, col 2)`.

b) what function of x, y does Rose want to maximize?

c) formulate Rose's maximization problem as an LP.

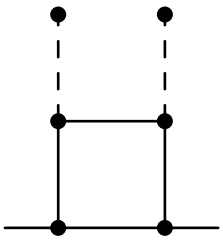
d) write the LP in sagemath format

e) using sagemath, solve Rose's problem.

f) repeat each of the above steps for Colin

g) what is the value of this matrix game?

8. Give the dyadic rational equivalent to this Hackenbush game. Show your work.



On Tuesday I will show you how to answer this next question.

9. Give the canonical form of each of these games. Which of them are equivalent to a dyadic rational?

```
{ -1, 0, 1 | 1/2, 3/4}
```

```
{ 1/2, 3/4 | -1, 0, 1 }
```

```
{ 7/4 | 3/4 }
```

```
{ 3/4 | 7/4 }
```

```
{ -3/8 | -7/8 }
```

```
{ -7/8 | -3/8 }
```

```
{ 5/2 | 5/4 }
```

```
{ 5/4 | 5/2 }
```