cmput 497/670 2024 homework 4

Explain all answers carefully.

1. Find the canonical form of the 4-pile nim game nim(15, 27, 14, 9).
   Hint: `1 1 1 1, 1 1 0 1 1, 1 1 1 0, 1 0 0 1 1.`
   Which theorems if any are you using in your answer?

2. Prove directly (without using any theorems) that the impartial game g with move options $\{*0, *1, *2, *4, *7\}$ equals the game $*3$.

3. Prove that a game k with move options $\{*k_1, *k_2, *k_3\}$ is equal to $*\text{mex}(k_1, k_2, k_3)$. This question is a special case of what theorem?

4. Prove by induction that the m×n chop position equals $*(m-1) + *(n-1)$.

5. a) Give the canonical form of the game g = chop(3×4) + bricks(5) + nim(5).
   b) If you play first on g, what move do you make?

6. When we think of hex as a combinatorial game, no moves are allowed after a player joins their two sides, and players do not necessarily alternate turns. In canonical form, give every combinatorial game equivalent to a hex position of a board with at most 3 rows and at most 3 columns. E.g. 0 is the canonical form of any position where a player has joined their two sides.

   a) 1×1 board: games 0 and $*$: why?

   b) 1×2 board: games 0, $*$, $\uparrow$, $\downarrow$: why? see below

   c) 2×2 board: all the above games (why?) as well as ???

   d) 2×2 board: ???

   e) 3×3 board: ???

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a game (in can'l form)          hex position

* = { 0 | 0 }         1x1 example         *
                                        o . o
                                          *

up = { 0 | * }         1x2 example          * *
                                          * . . o
                                             o o
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