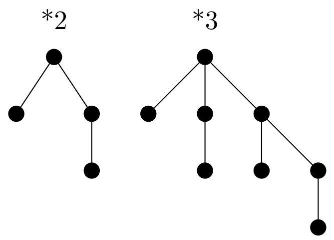
CMPUT 497/670 Homework 2

1

Group members: Owen Randall and Mahya Jamshidian. I understand that any discussion of this assignment outside my group members is plagiarism. I have not used any resources outside the class notes and textbook.

2

a) $*2 = \{*0, *1\}$, $*3 = \{*0, *1, *2\}$



b) Using the minimum exclusion principle (MEX), we can see the minimum nimber not in the set $\{*0, *1, *2, *5, *11\}$ is $*3$. Therefore: $\{*0, *1, *2, *5, *11\} = *3$.

c) Representing this game in canonical form and using MEX to simplify: $\{*0, *1, \{*1\}\} = \{*0, *1, *0\} = \{*0, *1\} = *2$

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$$a)\ *3 \rightarrow 011 \oplus *4 \rightarrow 100 \oplus *5 \rightarrow 101 = 010 \rightarrow *2$$

$$b)\ *7 \rightarrow 0111 \oplus *9 \rightarrow 1001 \oplus *14 \rightarrow 1110 \oplus *6 \rightarrow 0110 = 0110 \rightarrow *6$$

$$c)\ *19 \rightarrow 010011 \oplus *37 \rightarrow 100101 \oplus *28 \rightarrow 011100 \oplus *33 \rightarrow 100001 = 001011 \rightarrow *11$$

a) Take 2 stones from the first heap
b) Take 6 stones from the first heap (there are several correct moves here)
b) Take 5 stones from the third heap

a)	b)	c)
$a) * 1 \rightarrow 001$	$b) * 1 \rightarrow 0001$	$c) * 19 \rightarrow 010011$
$\oplus$	$\oplus$	$\oplus$
$*4 \rightarrow 100$	$*9 \rightarrow 1001$	$*37 \rightarrow 100101$
$\oplus$	$\oplus$	$\oplus$
$*5 \rightarrow 101$	$*14 \rightarrow 1110$	$*23 \rightarrow 010111$
$=$	$\oplus$	$\oplus$
$000 \rightarrow *0$	$*6 \rightarrow 0110$	$*33 \rightarrow 100001$
	$=$	$=$
	$0000 \rightarrow *0$	$000000 \rightarrow *0$

4

a)

$$\begin{aligned} &2 \times 3 \text{ Chomp } + 2 \times 4 \text{ Chop } + *5 \\ &= *4 + (*1 + *3) + *5 = *3 \end{aligned}$$

If we look at the nimber sum as in figure 1, we can see that for example a winning move could be to change Chop 2×4 to 2×1 with nimber $*1$.

```
1 0 0
0 1 0
1 0 1
-----
0 1 1
```

Figure 1: Question 4, Part a, nimber sum.

b)

$$\begin{aligned} &\text{Pick Up}(4) + 5 \times 3 \text{ Chop } + *7 \\ &= *1 + (*4 + *2) + *5 \\ &= *0 \end{aligned}$$

Based on the theorem, this position is *0, so it is a losing position. Thus, there is no winning move.

c)

$$\begin{aligned} &\text{Pick Up}(11) + 18 \times 24 \text{ Chop } + *20 \\ &= *2 + (*17 + *23) + *20 \\ &= *2 + (*1 + *16) + (*1 + *2 + *4 + *16) + *4 + *16 \\ &= *16 \end{aligned}$$

If we look at the nimber sum as in figure 2, we can see that, for example, a winning move could be to change $*20$ to $*4$.

```
0 0 0 1 0
0 0 1 1 0
1 0 1 0 0
---------
1 0 0 0 0
```

Figure 2: Question 4, Part c, nimber sum.

5

$a + b$ is odd if and only if $a \oplus b$ is odd.
$\implies$ ($a + b$ is odd): W.L.O.G assume that a is odd.
Then, b has to be even.
Note how the *xor* of these two has to be odd given than the right-most bit is $1 \oplus 0 = 1$.

$\impliedby$ ($a \oplus b$ is odd): W.L.O.G assume that a has the left most bit equal to 1, meaning that a is odd.
Then, b's right most bit has to be 0, making b even.
Note how the sum of a and b has to be odd.
Then we know that $a + b - a \oplus b$ is even.

For any bit at location i in a and b, we have $a_i + b_i \geq a_i \oplus b_i$.

1. $a_i = b_i$, then $a_i + b_i = a_i \oplus b_i = 0$.
2. $a_i \neq b_i$ and b_i, then $a_i + b_i = a_i \oplus b_i = 1$.

Then all the bits in $a \oplus b$ are equal to those in $a + b$.
Note how the left most bit of $a + b$ is at least equal to the left most bit of $a \oplus b$.
Then, $a + b \geq a \oplus b$.

6

Let F represent a full cell and E an empty one.
Note how in a row, consecutive empty cells can be replaced by a single empty cell.

$k = 0$:
Board : $|E| = *0$, since it is a P-position.

$k = 1$:
Board: $|F| = \{|E|\} = \{*0\} = *1$.

$k = 2$:
Board: $|F|F| = \{|F|, |E|\} = \{*0, *1\} = *2$.

$k = 3$:
Board: $|F|E|F| = \{|F|\} = \{*1\} = *0$.

$k = 4$:
Board: $|F|E|F|F| = \{|F|F|, |F|E|F|, |F|\} = \{*2, *0, , *1\} = *3$.

Board: $|F|F|F|F| = \{|F|F|, |F|E|F|, |F|F|F|, |F|E|F|F|\} = \{*2, *0, *3, *3\} = *1$.

$k = 5$:
Board: $|F|E|F|F|F| = \{|F|F|F|, |F|E|F|F|, |F|E|F|E|F|, |F|E|F|\} = \{*3, *3, *1, *0\} = *2$.
Board: $|F|F|E|F|F| = \{|F|F|, |F|E|F|F|\} = \{*2, *3\} = *0$.
Board: $|F|F|E|F|F| = \{|F|F|F|F|, |F|E|F|F|F|, |F|F|E|F|F|, |F|F|F|, |F|E|F|F|\} = \{*1, *2, *0, *3, *3\}$
$= *4$.

$k = 6$:
Board: $|F|F|F|F|F|F| = \{|F|F|F|F|F|, |F|E|F|F|F|F|, |F|F|E|F|F|F|, |F|F|F|F|, |F|E|F|F|F|, |F|F|E|F|F|\}$
$= \{*4, *0, *1, *1, *2, *0\} = *3$.
Board: $|F|E|F|F|F|F| = \{|\}F|F|F|F|, |F|E|F|F|F|, |F|E|F|E|F|F|, |F|E|F|F|E|F|, |F|E|F|F|\}$
$= \{*1, *2, *2, *2, *3, *1\} = *0$.
Board: $|F|E|F|E|F|F| = \{|F|E|F|F|, |F|E|F|E|F|, |F|E|F|\} = \{*3, *1, *0\} = *2$.
Board: $|F|F|E|F|F|F| = \{|F|E|F|F|F|, |F|F|E|F|F|, |F|F|E|F|E|F|, |F|F|F|, |F|F|E|F|\} = *1$.

Following the same step, the final nimber value for each k is:

- $k = 0 : *0$

- $k = 1 : *1$
- $k = 2 : *2$
- $k = 3 : *3$
- $k = 4 : *1$
- $k = 5 : *4$
- $k = 6 : *3$
- $k = 7 : *2$
- $k = 8 : *1$

7

a)
$[1|1] = \{\} = *0$
$[1|2] = \{[1|1]\} = \{*0\} = *1$
$[1|3] = \{[1|2]\} = \{*1\} = *0$
$[2|2] = \{[1|1]\} = \{*0\} = *1$
$[1|4] = \{[1|3], [2|2]\} = \{*0, *1\} = *2$
$[2|3] = \{[1|1], [1|2]\} = \{*0, *1\} = *2$
$[4|5] = \{[1|3], [2|2], [1|4], [2, 3]\} = \{*0, *1, *2, *2\} = \{*0, *1, *2\} = *3$

b)

$$\begin{array}{ccc} *3 \rightarrow 11 & \quad & *2 \rightarrow 10 \\ \oplus & & \oplus \\ *2 \rightarrow 10 & & *2 \rightarrow 10 \\ = & & = \\ 01 \rightarrow *1 & & 00 \rightarrow *0 \end{array}$$

Since $[1|4] = *2$ and $[2|3] = *2$, moving to either of these positions are winning moves. There are no other winning moves.

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a)

*1	*2	*1	*2	*1	*2
*0	*3	*0	*3	*0	*1
*1	*2	*1	*2	*3	*2
*0	*3	*0	*1	*0	*1
*1	*2	*3	*2	*3	*2
*0	*1	*0	*1	*0	*1

b)
For position (i, j), nimber is:

- $*0$ if i, j are both odd.
- $*2$ if i, j are both even.
- $*3$ if exactly one of i, j is odd and it is the larger number.
- $*1$ if exactly one of i, j is odd and it is the larger number.