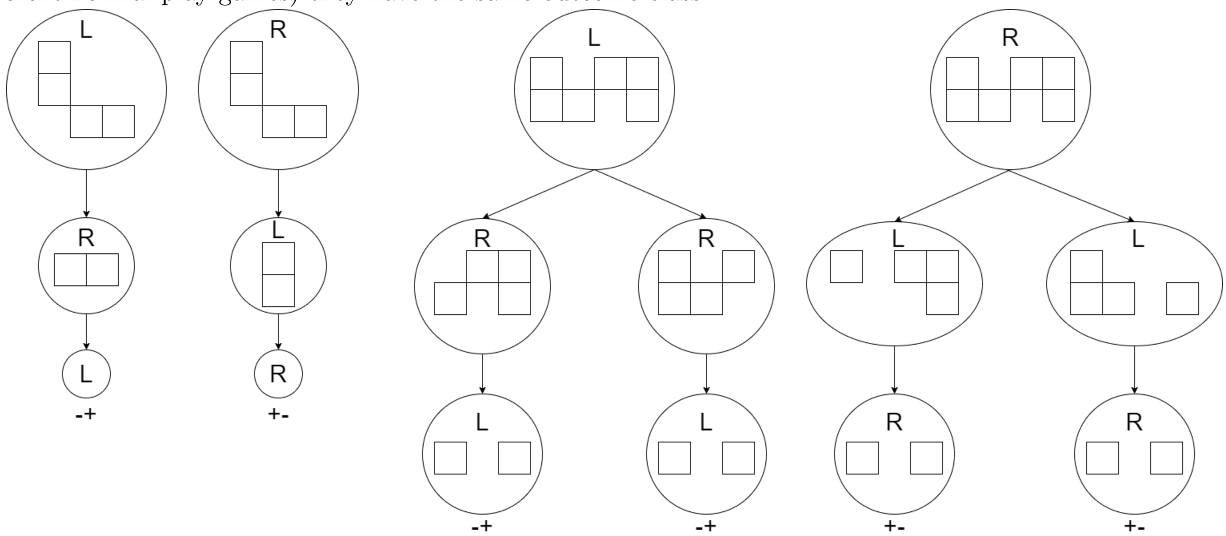
CMPUT 497/670 Homework 2

1

Group members: Owen Randall and Mahya Jamshidian. I understand that any discussion of this assignment outside my group members is plagiarism. I have not used any resources outside the class notes and textbook.

2

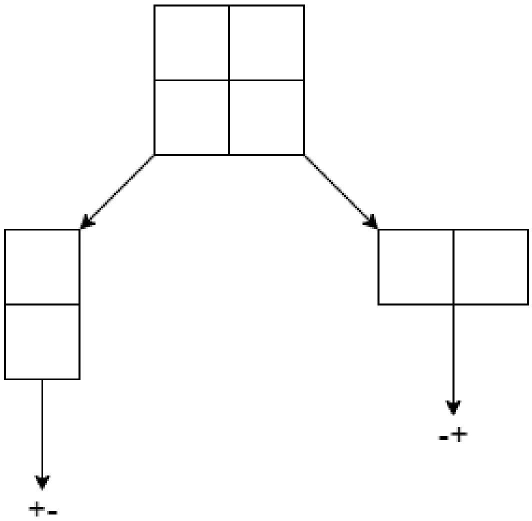
Two positions are equivalent if when summed with any other arbitrary position (possibly of entirely different normal play games) they have the same outcome class.

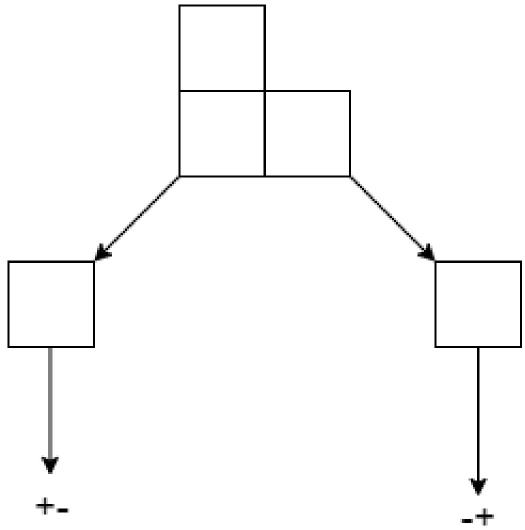


The above two games are P-positions as shown in the game trees, and so are equivalent. Looking at the game trees, we can see that they are not isomorphic.

3

Both of the game trees, as shown below, are N. This is because regardless of who is the first player, they will have winning strategy (Any move wins the game for the first player).





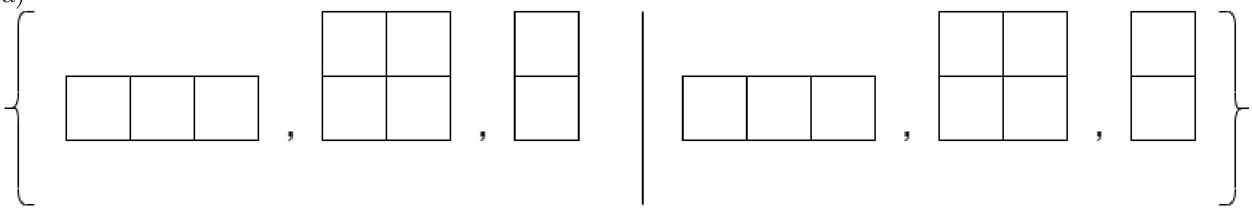
Let K be a another game which is the same as H, i.e. 3 cells in an L shape. Note how the outcome class for $G+K$ is N since first player plays on G, second player has no choice but to play on K, and then the next turn the first player again plays on G and wins.

The outcome class for $H+K$ is P, since first player plays on either boards and the next player plays on the other one and wins.

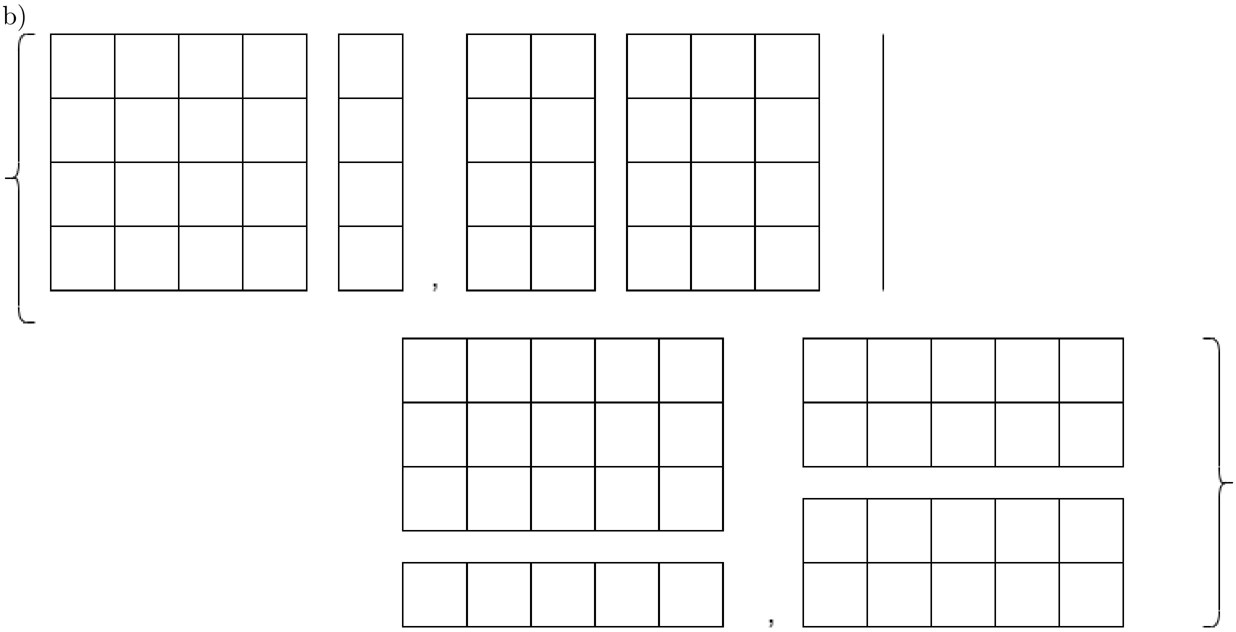
Since for the game K, $G+K \neq H+K$, we must have that G and H are not equivalent.

4

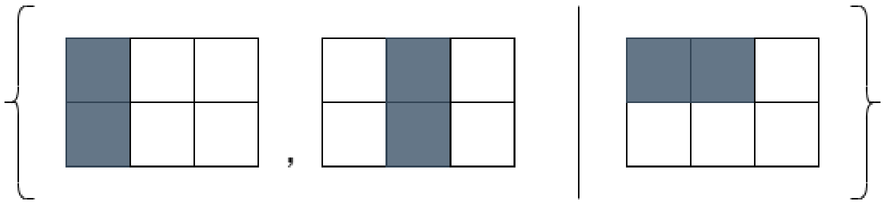
a)



b)

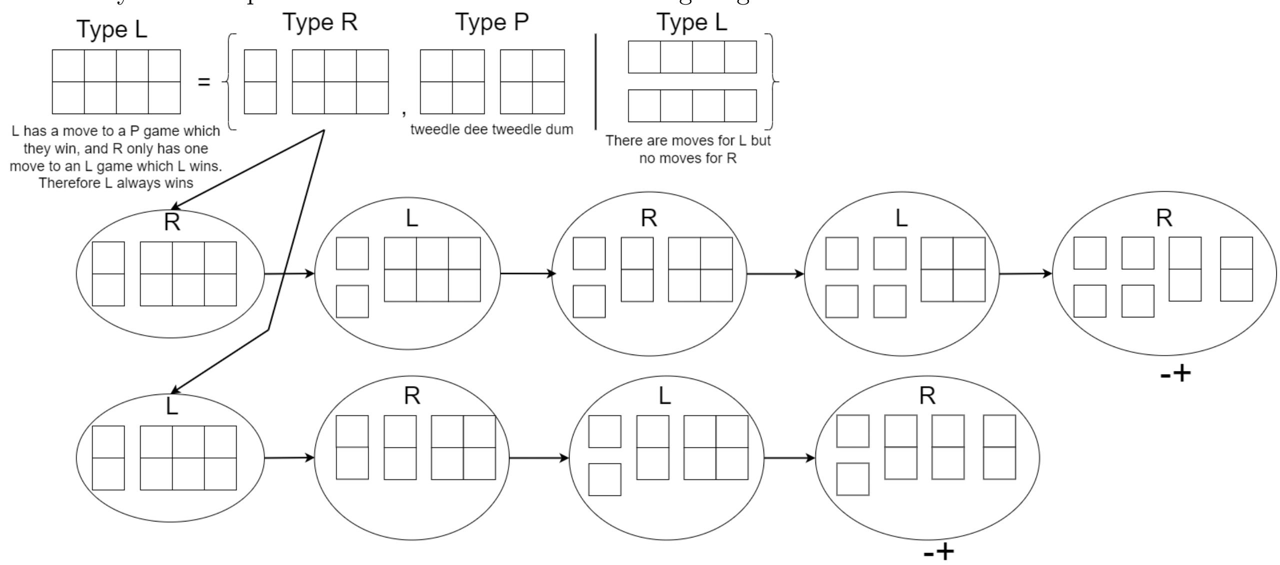


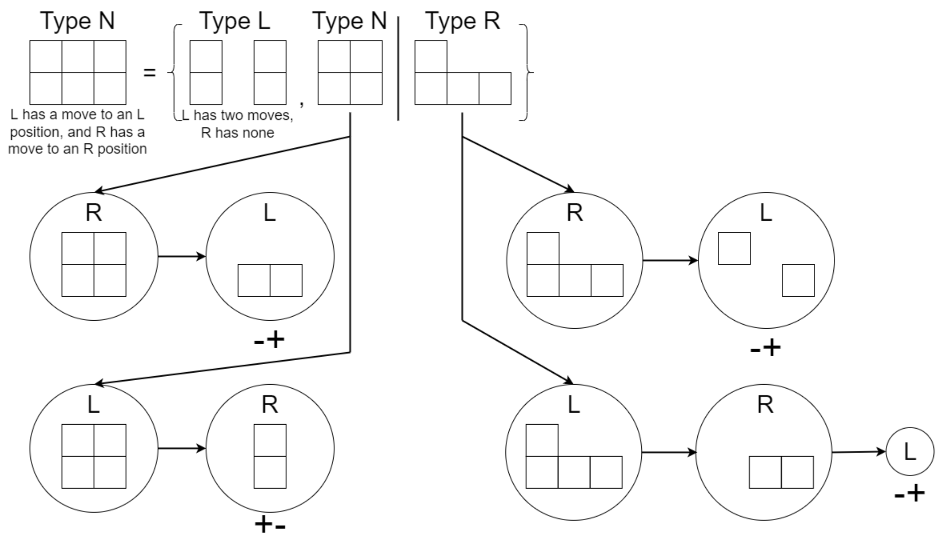
c)



5

Symmetrically identical positions are excluded in the following diagram:





6

a)
No matter who starts first, the first player plays on domineering game. Then, the next player has to play the brick.

Case 1) Second player picks up 1 brick: Then the first player picks up the other brick and wins.

Case 2) Second player picks up 2 bricks: Then the first player plays on domineering board and wins.

b)
The first player must pick 2 bricks.

Case 1) Second player plays brick with 1 or 2 pick up: Then the first player picks up the rest of the bricks. Now, the second player must cut. The result of their cut is a either a 1×3 or 3×1 block. Then, the first player cuts on this block and creates a 1×2 or 2×1 block. The second player makes the only move possible and loses.

Case 2) Second player plays on the cut board: which results in the same block as before, leading to first player's win on this game. Then, the second player picks up either 1 or 2 bricks and the first player picks up the rest and wins.

7

a)
Inconclusive, as $\alpha + \beta$ and $\alpha^{'} + \beta$ have to be the same type for all β.

b)
$\alpha \neq \alpha^{'}$ as there exists β such that $\alpha + \beta \neq \alpha^{'} + \beta$.

c)
$\alpha = \alpha^{'}$ per lemma 2.16.

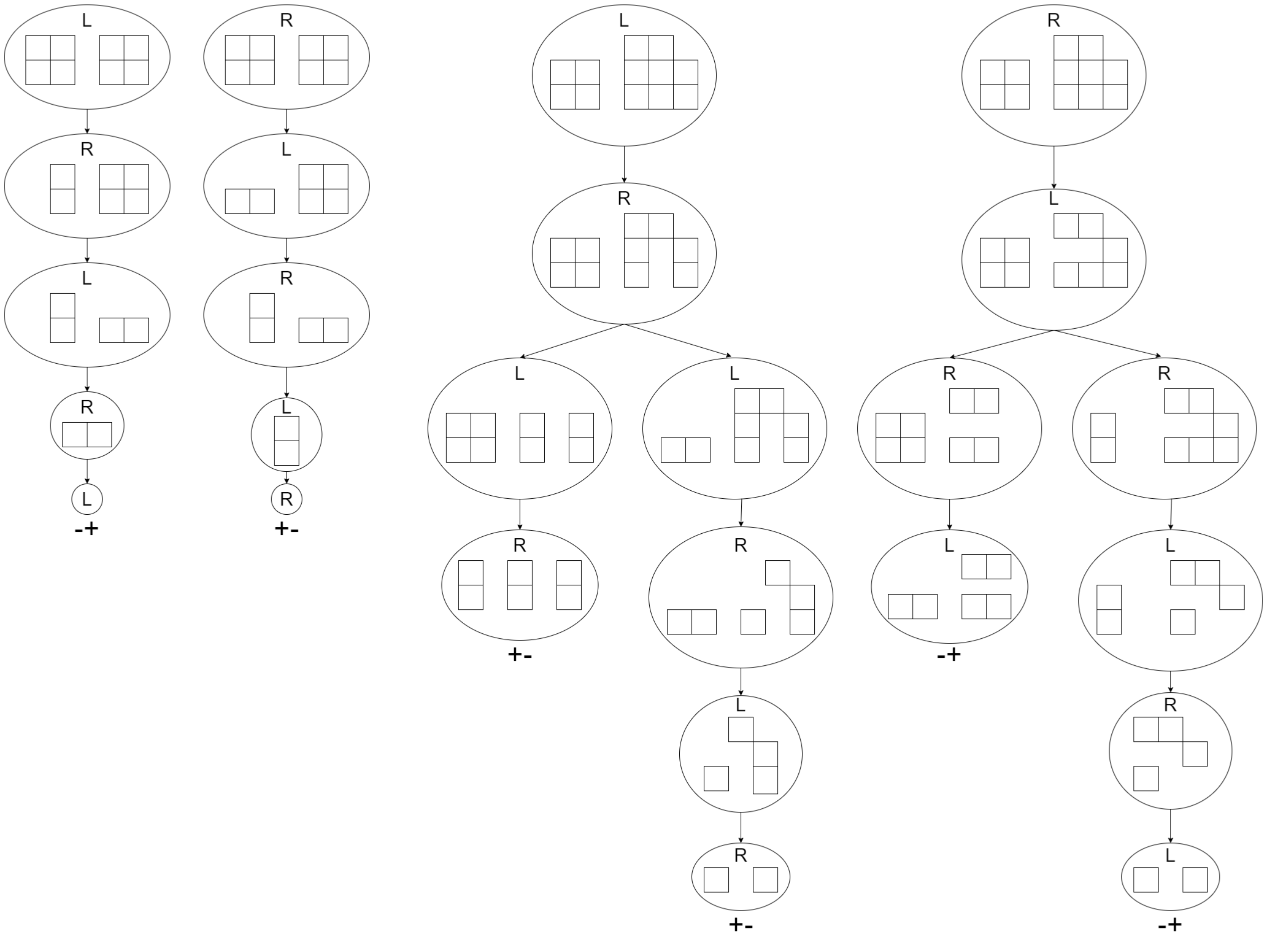
d)
$\alpha \neq \alpha^{'}$ as there exists β such that $\alpha + \beta \neq \alpha^{'} + \beta$.

e)
Inconclusive, as $\alpha + \beta$ and $\alpha^{'} + \beta$ have to be the same type for all β.

8

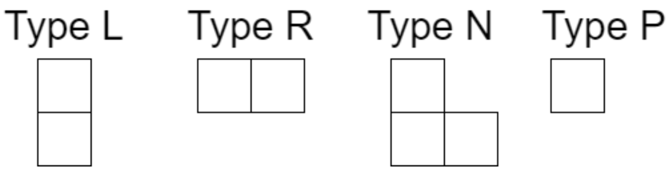
a) For game α, the first player can play any move and leave the opponent with no moves, making it a win for the first player.
For game β, L can play at the bottom of the middle column, leaving one move for R and two additional moves for L making it a win for L. R can play on the left side of the middle row, leaving one move for L and two additional moves for R making it a win for R.

b) The following diagram shows that $\alpha + \alpha$ is a P type game, and $\alpha + \beta$ is an N type game.



c) In part b) we have shown that $\alpha + \alpha$ is type P, and $\alpha + \beta$ is type N. Then by the definition of equivalence, since α behaves differently under summation compared to β, these two positions are not equivalent, i.e. $\alpha \not\equiv \beta$.

9



10

Let β be a 2×1 block.
It is trivial that $\alpha + \beta$ is P.

If R plays first, it either chooses one of the two bottom moves from either first or second blocks in $\alpha^{'} + \beta$. Then, L chooses either one 2×1 block from $\alpha^{'}$ or just β. Now R plays on the other block from $\alpha^{'}$ not chosen in the first pick. Then L makes a move with 1 or two options left an wins.

If L plays first, it has to choose one of the blocks from $\alpha^{'}$ or β itself. Either case, R can block L from picking at least one of the blocks from $\alpha^{'}$ with its move. Then, L plays the only option remaining and R wins in the next round.

Since $\alpha + \beta$ and $\alpha^{'} + \beta$ are both P, per lemma 2.16 α and $\alpha^{'}$ are equivalent.