497-670 2022 homework 1 solutions

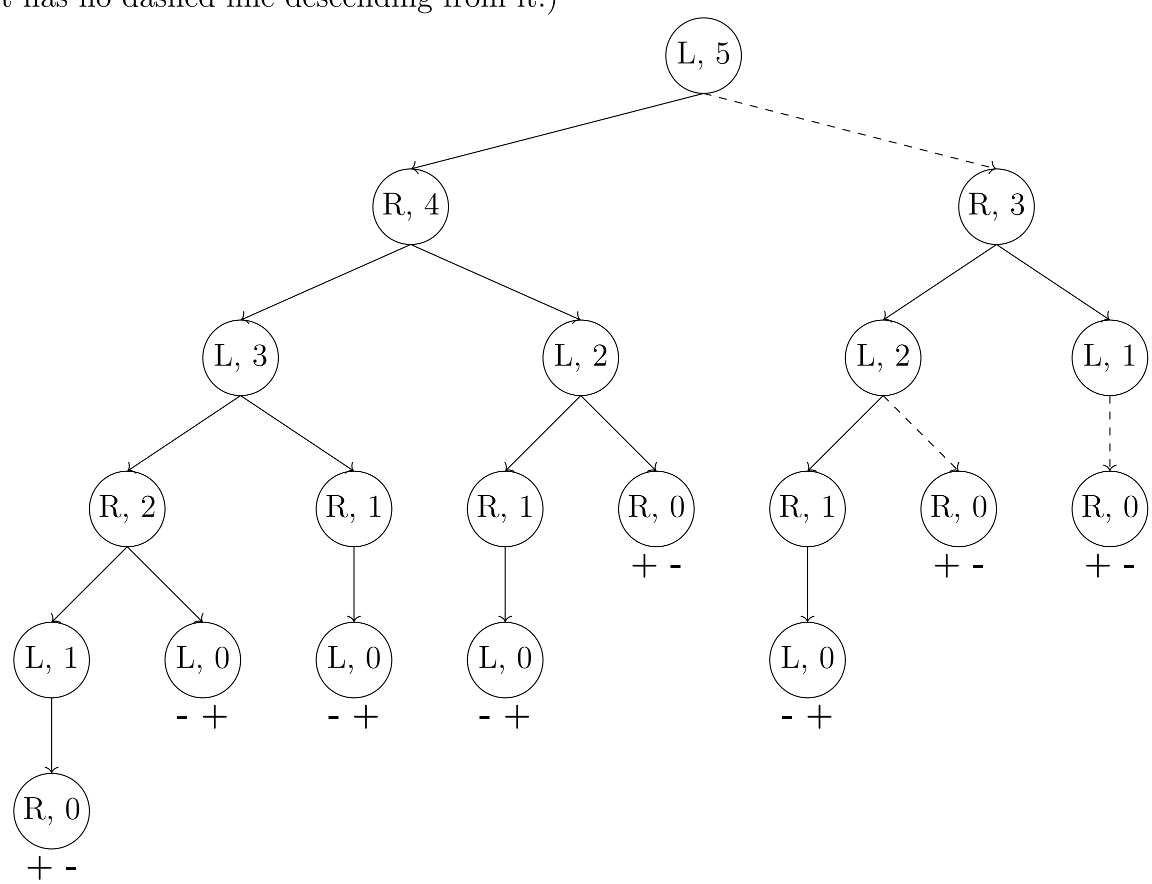
by Owen Randall and Mahya Jamshidian and RBH

1

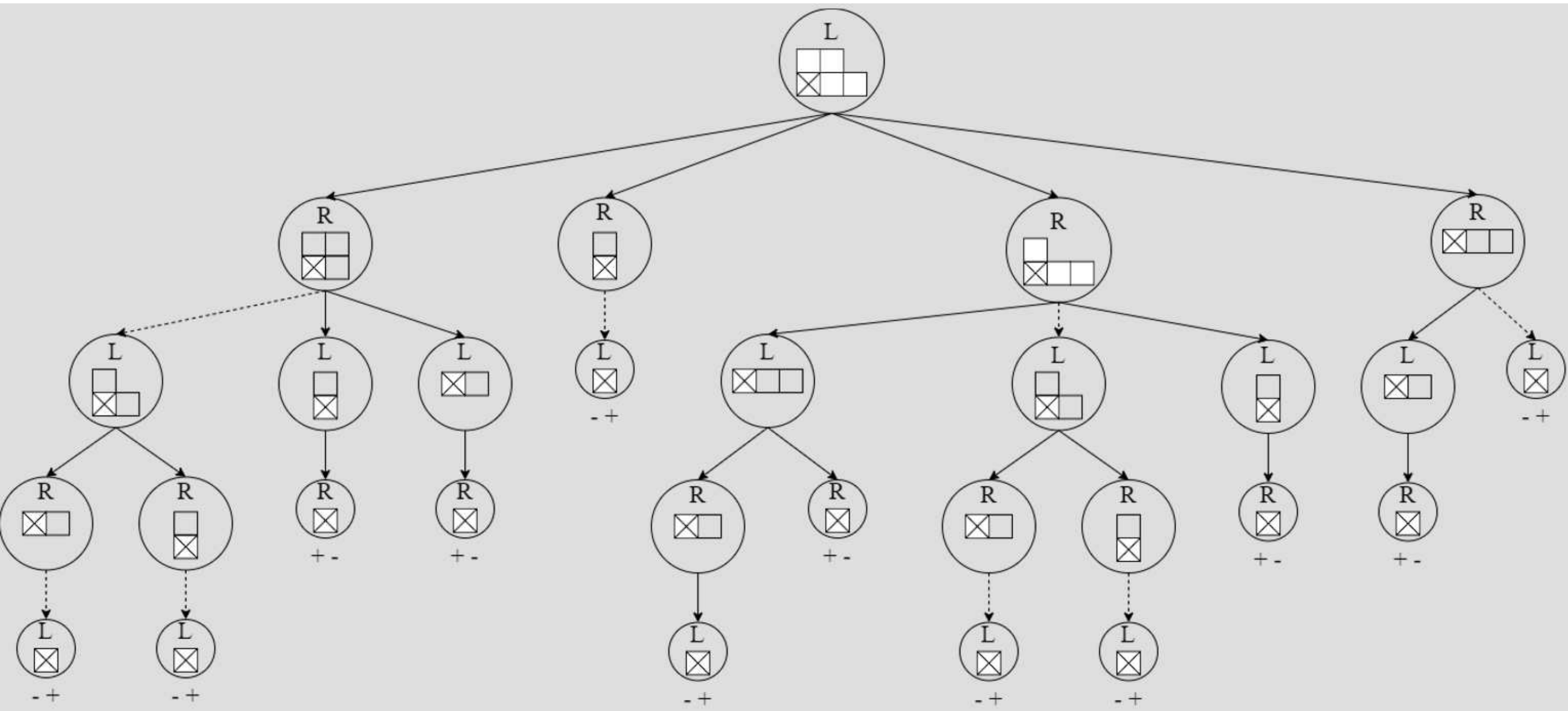
I understand that discussing this assignment with anyone outside of my group is considered plagiarism. No sources outside of the class material and assigned textbook were used.

2

a) The dashed lines of the game tree below gives a winning strategy for Louise (L). (To meet the textbook's definition of strategy, you also must include exactly one L-option for each L-node that has at least one child but has no dashed line descending from it.)



b) A winning strategy for Richard (R) can be seen in the dashed lines of the game tree. (Again, to meet the textbook's definition of strategy, you need to add a dashed line from each R-node that has a child but as yet has no dashed line.)



3

a) Strategies for Louise:

ABC	AB'C	AB'C'	ABC'
A'BC	A'B'C	A'B'C'	A'BC'
A"BC	A"B'C	A"B'C'	A"BC'

b) Strategies for Richard:

XY	XY'	XY"
X'Y	X'Y'	X'Y"

c) Drawing strategies for Louise: A'BC A'B'C A'B'C' A'BC' A"BC

d) Drawing strategies for Richard: X'Y' X'Y"

4

For a two-row chomp position such that the bottom row has length n, the top row has length m, and $m \leq n$, there exists a winning strategy for the first player for every value of n and m such that $n \neq m + 1$.
If $n = m + 1$, then there exists a winning strategy for the second player.

I will be using the notation (x, y) to denote a chomp position where the bottom row is length x and the top row is length y, e.g. $(1, 0)$ is the end of the game with only the poison square, $(2, 2)$ is a 2x2 square etc.

Proof:

Case 1: $n = m + 1$

We can denote this position as: $(m + 1, m)$.

Claim: There exists a winning strategy for the second player in the position $(m + 1, m)$ for $m \geq 0$.

Proof by induction:

Base case $m = 0$:

This is the game position $(1, 0)$ in which the first player cannot make any moves, so this position is a second player win.

Inductive hypothesis: assume there exists $k \geq 0$ such that for all $0 \leq m \leq k$ there exists a second player winning strategy for the position $(m + 1, m)$.

Prove that the position $(k + 2, k + 1)$ has a second player winning strategy:

Case 1.1: The first player plays on the top row

Then the first player will have taken $1 \leq i \leq k + 1$ blocks from the top row, making the new game position: $(k + 2, k + 1 - i)$.

The second player may then take i blocks from the bottom row, making the new game position $(k + 2 - i, k + 1 - i)$.

Since $i \geq 1$, the game position can be rewritten as $(m + 1, m)$ where $m \leq k$, for which there exists a second player winning strategy by the induction hypothesis.

Therefore a second player winning strategy exists for every position in this case.

Case 1.2: The first player plays on the bottom row

Then the first player will have taken $1 \leq i \leq k + 1$ blocks from the bottom row, making the new game position: $(k + 2 - i, k + 1 - (i - 1)) = (k + 2 - i, k + 1 - i + 1) = (k + 2 - i, k + 2 - i)$.

The second player may then take 1 block from the top row, making the new game position $(k+2-i, k+1-i)$.

Since $i \geq 1$, the game position can be rewritten as $(m+1, m)$ where $m \leq k$, for which there exists a second player winning strategy by the induction hypothesis.

Therefore a second player winning strategy exists for every position in this case.

Cases 1.1 and 1.2 cover every possible first player move when $n = m+1$, therefore there exists a second player winning strategy when $n = m+1$

Case 2: $n \neq m+1$

For the sake of clarity in this case, we assume without loss of generality that L is the first player and R is the second player.

Case 2.1: $n = m$

This can be written as the game position (m, m), or $(x+1, x+1)$ where $x+1 = m$.

L can take one stone from the top row, making the new game $(x+1, x)$, which R must now play on.

As shown in Case 1, this new position is a win for the second player, which in this position is L. Therefore L (the first player) has a winning strategy for every position in this case.

Case 2.2: $n > m$

This can be written as the game position $(m+x, m)$, where $x = n-m$.

L can take $x-1$ stone from the bottom row, making the new game $(m+x-(x-1), m) = (m+x-x+1, m) = (m+1, m)$ which R must now play on.

As shown in Case 1, this new position is a win for the second player, which in this position is L. Therefore L (the first player) has a winning strategy for every position in this case.

Cases 2.1 and 2.2 cover every possible position when $n \neq m+1$, therefore there exists a first player winning strategy when $n \neq m+1$.

Cases 1 and 2 cover every possible position where $m \leq n$, so we are done.

5

We prove this using induction on the height of the game tree.

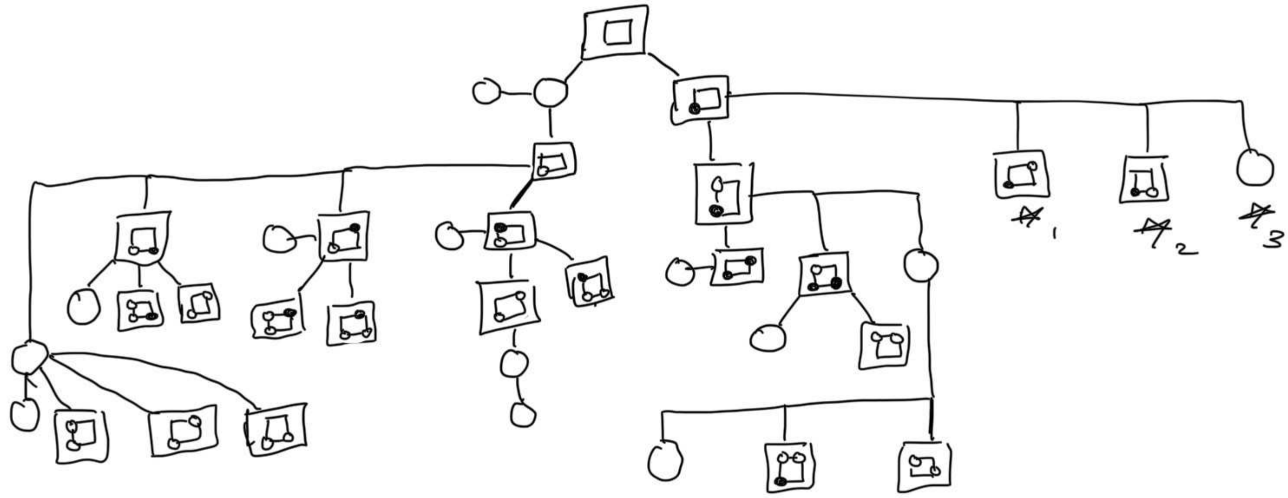
Base case) Height = 0: A tree of height 0 is already decided. So, either it is (+-) which is L's winning strategy, or (-+) being R's winning strategy.

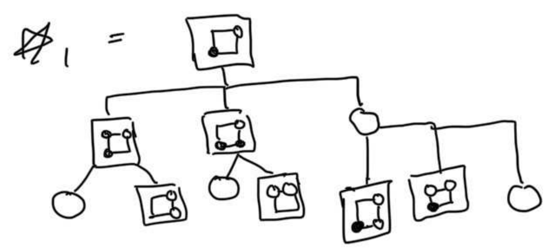
Induction hypothesis) Height is k.

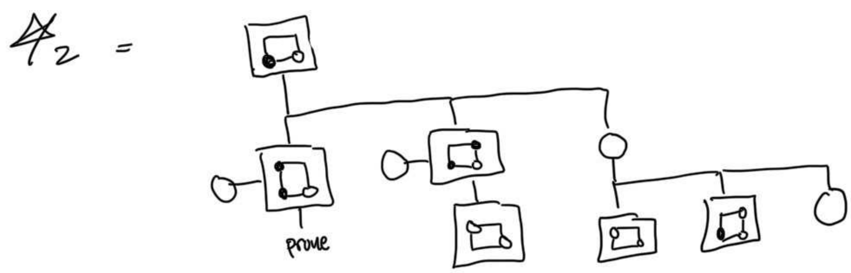
Consider a game tree of height $k+1$. Assume L goes first: Since all the sub-trees rooted at children of root are of height k or less, one can determine the result of each sub-tree which can be either (+-) and (-+). Then, if any of the sub-trees have (+-) label, then the whole game is (+-). Else, if all the sub-games are labeled (-+), then the game is (-+).

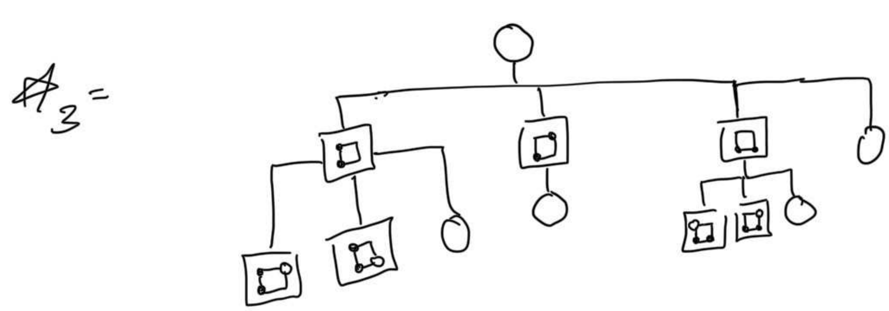
6

Here is the game tree: you only needed to draw the top 5 levels of this tree.





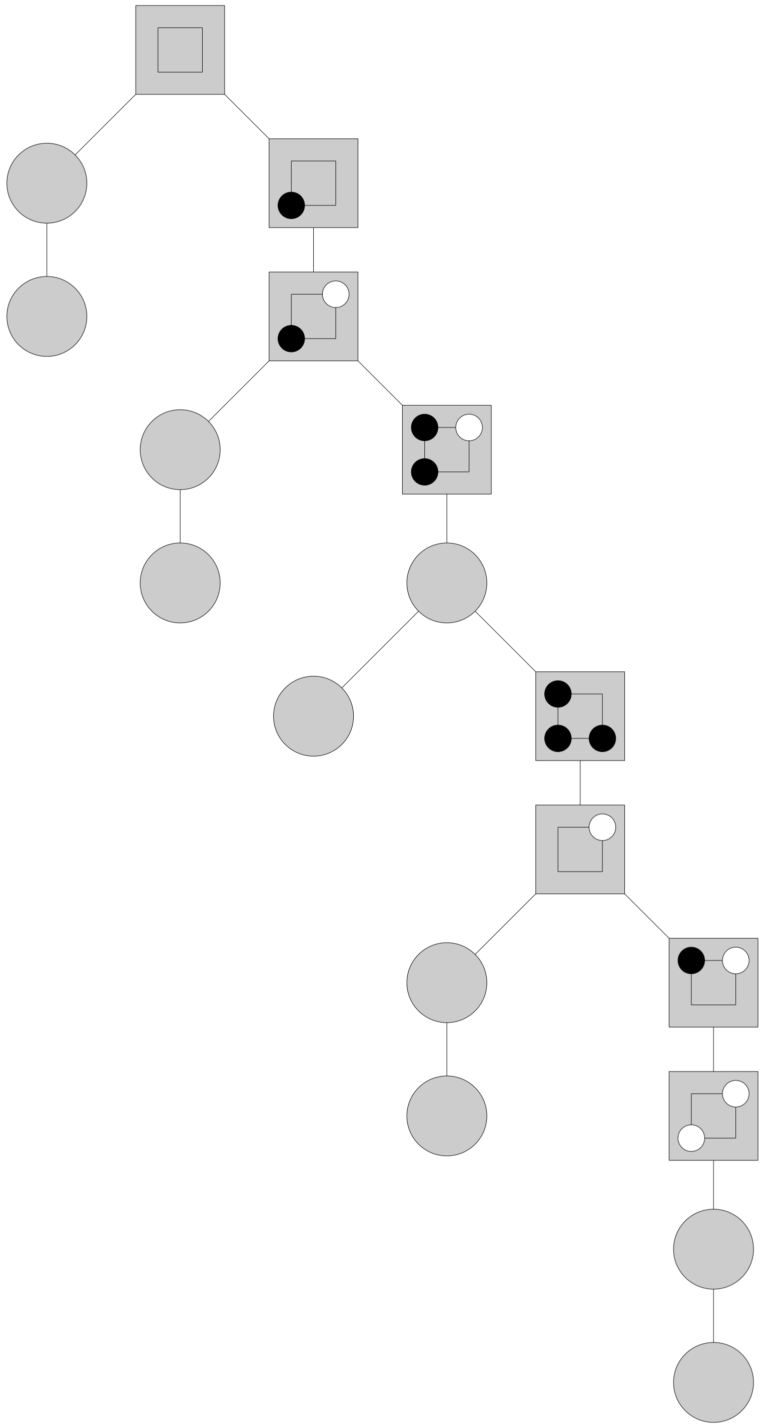




7

(a) 8.W[b_2] would repeat the position after 4.W[b_1], violating the positional superko rule.

(b) Tree for 7b below.



8

i)

Black chains: 5	White chains: 6
Black stones: 26	White stones: 25
Black territory : 2	White territory: 3

ii) i5. This move increases white's score by 4 (1 stone + 3 new territory), where as any other move would only increase white's score by at most 3.

iii) White by 65. There is one "safe" black block in the bottom right corner of the board, i.e. a block with two eyes which cannot be captured as any attempt by white to play in one of those eyes would be suicide as there is another eye providing the block a liberty. Since suicide is not allowed, this block cannot be captured. This black block has of 6 stones and surrounds 2 black territory points. White can play to control the rest of the board (as there are no other safe black blocks) which has 9x9-8=73 points, so then the best score for white would be 73-8 = 65.