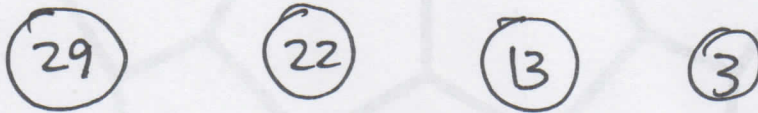


NIM GAME

IS THIS A P-PSN OR N-PSN ?



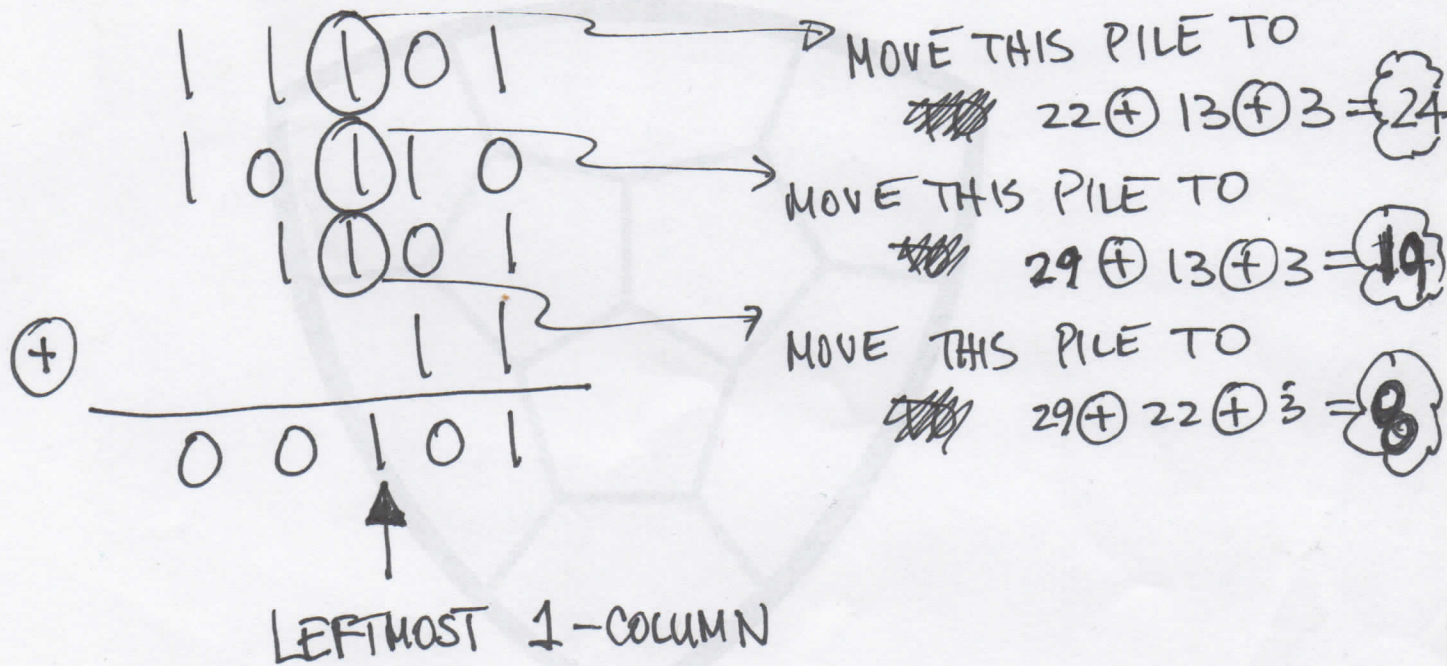
ANSWER:

$$29 \oplus 22 \oplus 13 \oplus 3 =$$

$$11101_2 \oplus 10110_2 \oplus 1101_2 \oplus 11_2 = 101_2 \neq 0_2$$

so N-PSN

ALL WINNING MOVES ?



CLAIM

EVERY (POSSIBLY MULTI-PILE)

NIM POSITION IS EQUIV. TO SOME NUMBER.

PROOF

(SKETCH)

$$*29 + *22 + *13 + *3 \equiv$$

$$*(29 \oplus 22 \oplus 13 \oplus 3) = *5$$

WHY?

$$*29 + *22 + *13 + *3 + *? \equiv \emptyset$$

IFF

$$? = 29 \oplus 22 \oplus 13 \oplus 3$$



$(p_1, p_2, p_3, \dots, p_t)$

BECAUSE A NIM POSITION IS A P-POSITION

IFF

$$p_1 \oplus p_2 \oplus p_3 \oplus \dots \oplus p_t = \emptyset$$

MEX THEOREM: Let $G = \{ *a_1, *a_2, \dots, *a_k \}$
 WHERE EACH $*a_j$ IS A NUMBER. THEN $G \equiv *a_m$,
 WHERE $m = \text{MEX} \{ \cancel{*}a_1, a_2, \dots, a_k \}$,

E.G. $\text{MEX} \{ *0, *1, *4, *7 \} = *2$

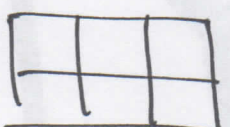
$\text{MEX} \{ *1, *2, *5 \} = *0$

2022
 FEB 15
 P3

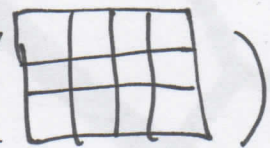
PROOF IN TEXT

E.G. PICKUP BRICKS (n) $\equiv *(n \text{ MOD } 3)$
BRICKS

E.G. CHOP ($m \times n$ RECTANGLE) $\equiv *(m-1) + *(n-1)$

E.G. CHOP () $\equiv *1 + *2$

NOTICE $*1 + *2 = \{ *2, *1+*1, *1 \}$
 $= \{ *2, *0, *1 \} = *3$

E.G. CHOP () $\equiv *2 + *3$
 $\equiv *(2 \oplus 3)$
 $\equiv *1$



APPLYING MEX THEOREM

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WHAT NUMBER IS THIS?

$$\begin{aligned}
 & \underline{\text{CHOP}(4 \times 5) + \text{BRICKS}(7) + \text{NIM}(4)} \equiv \\
 & *3 + *4 + *1 + *4 \equiv \\
 & *(3 \oplus 4) + *1 + *4 \equiv \\
 & *7 + *1 + *4 \equiv \\
 & *(7 \oplus 1 \oplus 4) \equiv *2
 \end{aligned}$$

$$\begin{array}{r}
 11 \\
 \underline{101} \\
 111
 \end{array}$$

$$\begin{array}{r}
 111 \\
 \underline{100} \\
 010
 \end{array}$$

HOW TO PLAY THIS GAME?

$$\begin{array}{r}
 101 \\
 \underline{100} \\
 010 \\
 \uparrow
 \end{array}$$

MOVE THIS PILE TO $1 \oplus 4 = 5$

ONLY WINNING MOVE

SO ON THE *7 GAME (4x5 CHOP),
MOVE TO CHOP POSITION WITH VALUE *5

$$\text{CHOP}(4 \times 5) = \left\{ \begin{array}{ccccccc}
 3 \times 5 & 2 \times 5 & 1 \times 5 & 4 \times 4 & 4 \times 3 & 4 \times 2 & 4 \times 1 \\
 \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\
 *6 & *5 & *4 & *0 & *1 & *2 & *3
 \end{array} \right\}$$