

CLOBBER: DOES $G = H$?

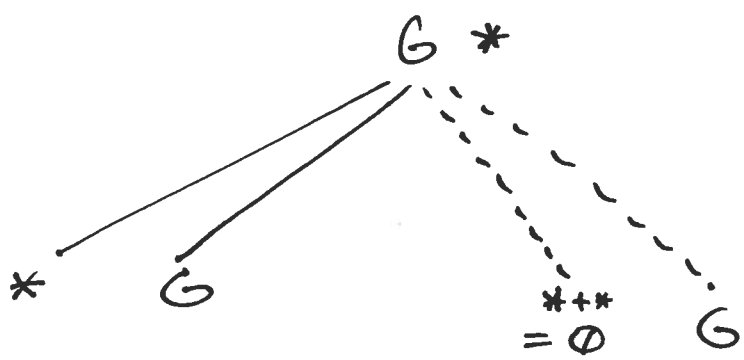
LECTURE
TU
FEB 25
2020

RECALL DEF'N = : $G=H$ IFF
 $\forall X, G+X, H+X$ SAME O.C.

USEFUL THM: $G=H$ IFF $G+H=0$
IS A P-PSN

$G+H$ TREE ?

$H = *$
SO $-H = *$
SO $G+H = G+*$



← NOTICE:
ONE PLAYER (R)
HAS OPTION TO 0,
SO $G*$ NOT
A P-PSN

SO $G+H \neq 0$

SO $G \neq H$



SOME NAMES

TREE

NOT'N

①



$\{1\}$

1



$\{0|1\}$

-1



$\{1|0\}$

*



$\{0|0\}$

↑



$\{0|*\}$

↓

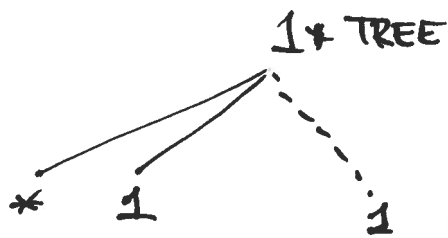


$\{*\|0\}$

1*

MEANS

1 + *



$$G = 1* = \{1, * | 1\}$$

NOTICE

LEFT WOULD ALWAYS PREFER OPTION 1
INSTEAD OF " " *

SO WE SUSPECT $G \stackrel{?}{=} \{1|1\} = H$

EXERCISE :

PROVE $G = H$,

i.e. PROVE $G + -H = 0$

EXERCISE

FIND TWO ~~GAMES~~ UNEQUAL GAMES
IN SAME OUTCOME CLASS.

NOTICE : ANY TWO P-PSNS ARE BOTH $= 0$,
SO EQUAL.

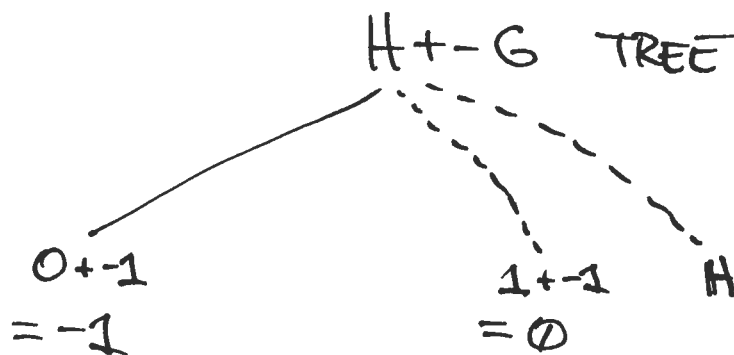
• WHAT ABOUT L-PSNS ?

• ARE THESE EQUAL ?



$$H = G \quad \text{IFF} \quad H + -G = 0$$

$$\boxed{-G = -1}$$



NOTICE:
R HAS 0 OPTION,
SO $H + -G$ NOT
A P-PSN

$$\text{SO } H + -G \neq 0$$

$$\text{SO } H \neq G$$



RECALL " $= \emptyset$ " MEANS "IS A P-PSN".

EXTEND THIS TO $<, >, ||$

INCOMPARABLE
OR FUZZY.

" $< \emptyset$ " IS A R-PSN

" $> \emptyset$ " IS A L-PSN

" $|| \emptyset$ " IS A N-PSN

SIMILARLY

$G < H$	IFF	$G + -H < \emptyset$
$G > H$	"	" $> \emptyset$
$G H$	"	" $ \emptyset$

~~By~~ $<, >, =$ ALL TRANSITIVE

E.G. IF $A < B$ AND $B < C$

THEN $A < C$

E.G. IF $A \leq B$ AND $B \leq C$

THEN $A \leq C$

$||$ NOT NEC. TRANSITIVE.

EXERCISE FIND A, B, C S.T. $A || B, B || C$ BUT $A \not|| C$.

EXERCISE SHOW $||$ COMMUTATIVE.

1-PILE NIM GAME

# STONES	TREE	GAME NAME	
0	•	0	}
1	• / \	*	
2	• / \ / \	*2	
3	• / \ / \ / \	*3	
n	• / \ / \ / \ / \	*n	

THESE GAMES ARE CALLED NUMBERS

WARNING!

*~~n~~ IS ABOVE NUMBER

n*~~is~~ IS GAME ~~n~~ + *

E.G.

$$*1 = *$$

$$1* = 1 + *$$

$$*2 = \{0, * \mid 0, *\}$$

$$2* = 2 + *$$

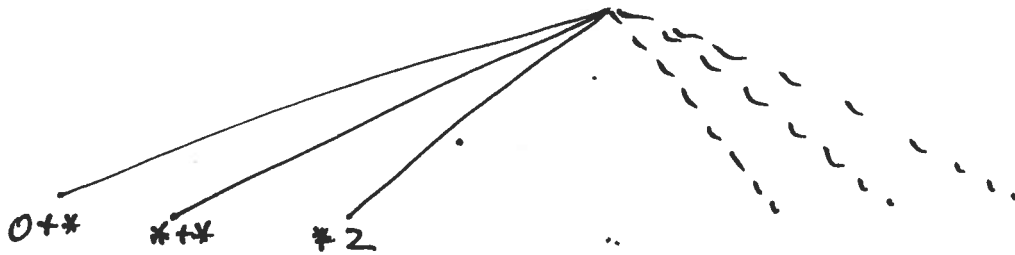
DOES $*2 = *$?

$$*2 = * \quad \text{IFF} \quad *2 - * = 0$$

$$\text{IFF} \quad *2 + * = 0$$

SINCE $-* = *$

$*2 + *$ TREE



L HAS 0 OPTION, SO $*2 + * \neq 0$

SO $*2 \neq *$



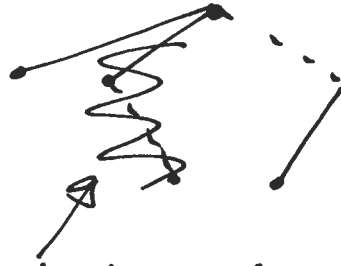
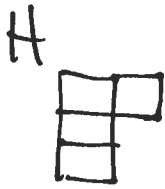
VALUE

DOM.



GAME 1

DOM



L WOULD NEVER ~~PREFER~~ PREFER THIS OPTION

~~THIS~~ GAME H IS L-POSN.

DOES IT HAVE SOME "VALUE"?

NOTICE

$$H + H - G = 0$$

$$\text{so } \text{val}(H) + \text{val}(H) - 1 = 0$$

$$\text{so } \dots \text{val}(H) = \frac{1}{2} ?$$

EXERCISE:

PROVE $H+H-G$ IS P-PSN

THIS WOULD MAKE SENSE.

IN FACT, WE CALL

THIS GAME



$\frac{1}{2}$

!!