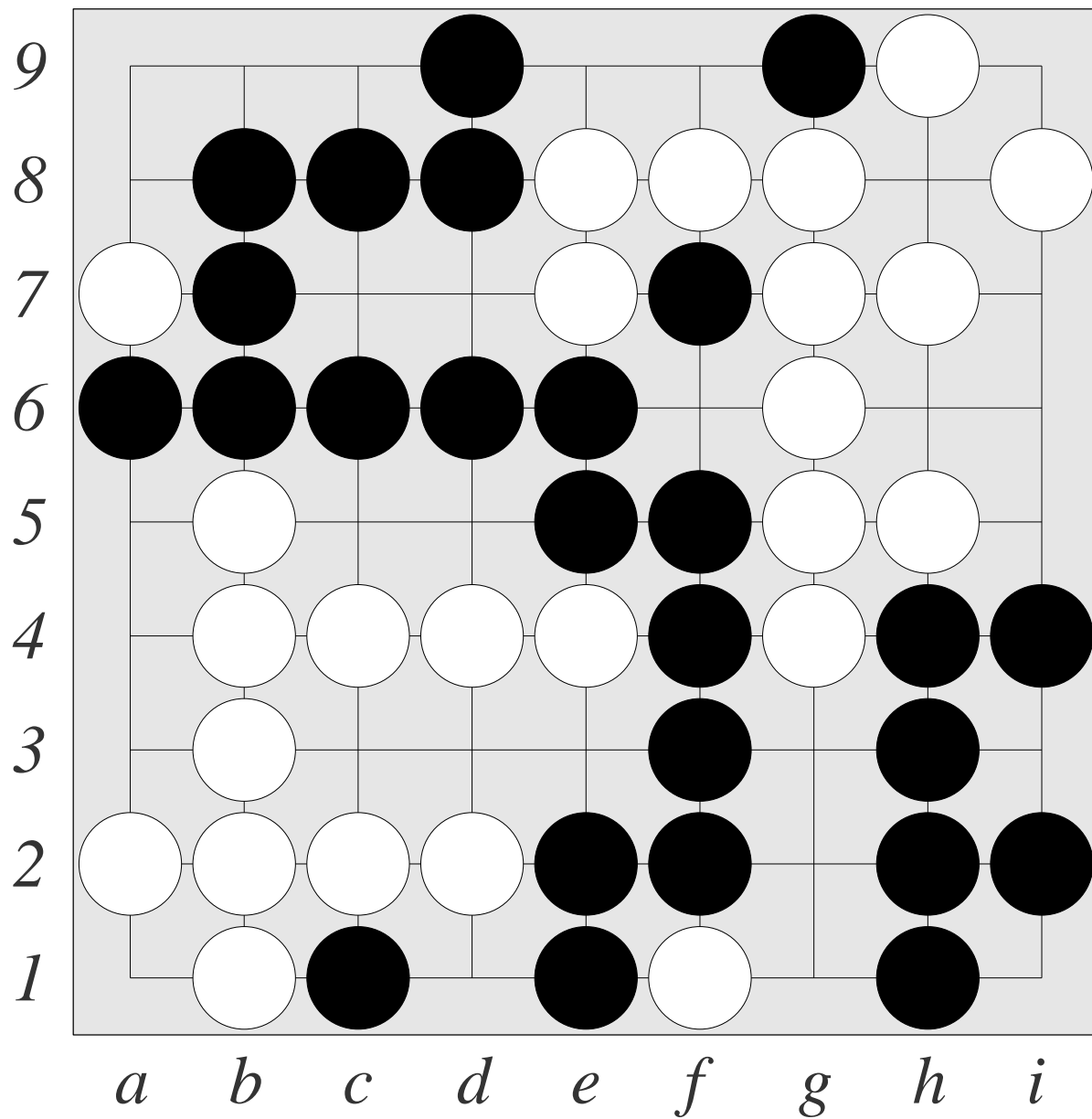


- our story so far ...
- comb GT and classical GT
- comb GT
- normal play games
- normal play games: sums
- why sums?
- because many games break down into sums
- e.g. go

cf. Berlekamp and Wolfe, *Mathematical Go: chilling gets the last point*



- clobber

x o x o x o x L (x) to play

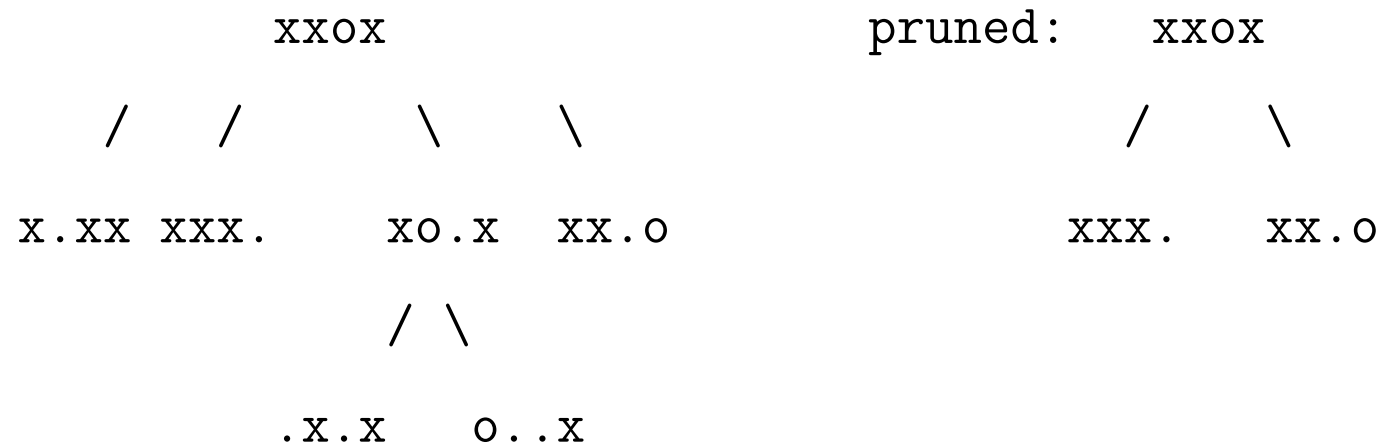
x o . x x o x R (o) to play

o . . x x o x x to play

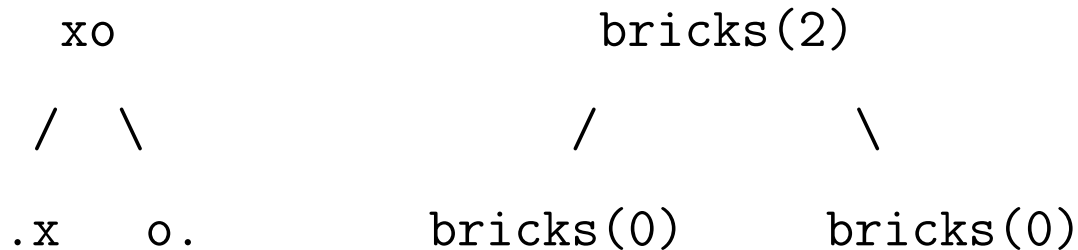
o . . x . x x o has no move: x wins

- combinatorial game as a tree
 - as usual, show all possible continuations of play ...
 - game might be summand: **players do not necessarily alternate turns**
 - usually, we prune inferior moves
 - left options are left branches
 - right options are right branches

e.g. game tree for clobber(xxox)



game trees for clobber(xo) and bricks(2)



- assume that each player always plays for their best outcome
- then we can sometimes prune trees
- trees for pruned-clobber(xxox), clobber(xo), bricks(2) are isomorphic
- we can treat clobber(xxox), clobber(xo), bricks(2) the same

- L R N P outcome classes: sums?
- below, for each sum, give a game for as many categories as you can
- use games we have seen: domineering, pickup bricks, clobber, ...
- if some outcome classes are not possible, prove why
- e.g. prove that the sum of a game in N and a game in P must be in N
- e.g. find a game A in L and B in R whose sum is in L
- e.g. prove that, when 1st player, L can win sum of L-game and N-game

- warmup: find the simplified game tree for $\text{clobber}(\text{xxo}) + \text{clobber}(\text{ox})$
- what does the simplified tree tell us about outcome sums?
- hint: this sum is $\text{clobber}(\text{xxo.ox})$

xxo.ox

pruned

xxo.ox

/ / \ \

/ \

x.x.ox xxo.x. xo..ox xxo..o

xxo.x. xo..ox (same as ...)

^ ^

/ \

| |

x.x.x. xo..x.

better for x better for o

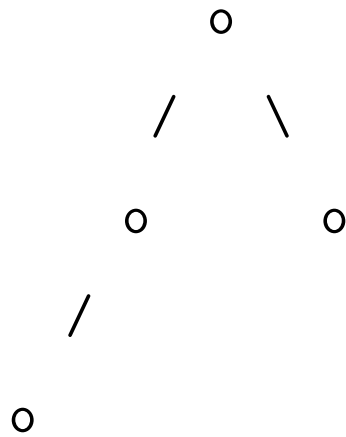
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.x..x.

- consequence?
- clobber(xxo) is in outcome class L
- clobber(ox) is in outcome class N
- sum is in outcome class N
- so sum of game in L and game in N can be in N
- also: as first player, L can win sum of any L-game and N-game (exercise: prove)
- conclusion: set of possible outcome for L-game + N-game is $\{L,N\}$

- consider the game X whose tree is below
- each o is a node
- what outcome class is X in?
- what outcome class is $X+X$ in?

game G



L

R

N

P

L L? R? N? P? ? ? ? ? ? ? ? ? ? ? ? ? ? ?

R ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?

N ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?

P ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?

L

R

N

P

L

L

L R N P

L N

L

R

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R N

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L R N P

N

P

P