CMPUT 497/670 Homework 5

1

Group members: Owen Randall and Mahya Jamshidian. I understand that any discussion of this assignment outside my group members is plagiarism. I have not used any resources outside the class notes and textbook.

2

a)
Note how the first column is dominated by the second and third column, so we can eliminate that. Also, [43] as a row dominates [20], and hence, the latter is eliminated. Then, the game value is 3 as C picks the third column and R picks the first row.

b)
The second column is dominated by the third and can be eliminated. Next, row [63] dominates the other two and hence, we can eliminate the first and second row. Then, the players are left with [63] and as C picks the third column and R picks the third row, the game value for C is 3.

c)
The first and third column dominate the second and hence we can remove it and get the following as the game matrix : $\begin{bmatrix} 5 & 4 & 7 \\ 1 & 3 & 0 \\ 7 & 5 & 6 \end{bmatrix}$. But note how the second row is dominated by the other two and hence, is eliminated. Then we have: $\begin{bmatrix} 5 & 4 & 7 \\ 7 & 5 & 6 \end{bmatrix}$. However, the second column dominates the other two and at last, R picks row 3 and C picks column 3 and the game value for C would be 5.

3

Let the notation (x, y) mean that one unit is deployed to x and one unit is deployed to y. a)

	(1,1)	(1,2)	(1,3)	(2,2)	(2,3)	(3,3)
(1,1)	0	-2	-3	-2	-5	-3
(1,2)	2	0	-1	1	-2	0
(1,3)	3	1	0	2	-1	-2
(2,2)	2	-1	-2	0	-1	-1
(2,3)	5	2	1	1	0	-1
(3,3)	3	0	2	1	1	0

b)

	(1,1)	(1,2)	(1,3)	(2,2)	(2,3)	(3,3)
(1,2)	2	0	-1	1	-2	0
(1,3)	3	1	0	2	-1	-2
(2,2)	2	-1	-2	0	-1	-1
(2,3)	5	2	1	1	0	-1
(3,3)	3	0	2	1	1	0

	(1,1)	(1,2)	(1,3)	(2,2)	(2,3)	(3,3)
(1,3)	3	1	0	2	-1	-2
(2,2)	2	-1	-2	0	-1	-1
(2,3)	5	2	1	1	0	-1
(3,3)	3	0	2	1	1	0

	(1,1)	(1,2)	(1,3)	(2,2)	(2,3)	(3,3)
(1,3)	3	1	0	2	-1	-2
(2,3)	5	2	1	1	0	-1
(3,3)	3	0	2	1	1	0

	(1,2)	(1,3)	(2,2)	(2,3)	(3,3)
(1,3)	1	0	2	-1	-2
(2,3)	2	1	1	0	-1
(3,3)	0	2	1	1	0

	(1,3)	(2,2)	(2,3)	(3,3)
(1,3)	0	2	-1	-2
(2,3)	1	1	0	-1
(3,3)	2	1	1	0

	(2,2)	(2,3)	(3,3)
(1,3)	2	-1	-2
(2,3)	1	0	-1
(3,3)	1	1	0

	(2,3)	(3,3)
(1,3)	-1	-2
(2,3)	0	-1
(3,3)	1	0

	(3,3)
(1,3)	-2
(2,3)	-1
(3,3)	0

	(3,3)
(2,3)	-1
(3,3)	0

	(3,3)
(3,3)	0

c) Both players should play (3,3) as this is a dominant strategy compared to every other pure strategy. This is the only move in which guarantees at least a draw.

4

a)
Row 2 and column 2 returns -2 for Rose and 2 for Colin.

b)

$\begin{bmatrix} 3 & 4 & 2 \end{bmatrix}. \begin{bmatrix} \frac{1}{5} \\ \frac{2}{5} \\ \frac{2}{5} \end{bmatrix} = \frac{3}{5} + \frac{8}{5} + \frac{4}{5} = \frac{15}{5} = 3$. Hence, Rose gets 3 and Colin gets -3.

c)

$\begin{bmatrix} \frac{2}{3} & \frac{1}{3} \end{bmatrix}. \begin{bmatrix} 2 \\ 7 \end{bmatrix} = \frac{4}{3} + \frac{7}{3} = \frac{11}{3}$. Then, Rose gets $\frac{11}{3}$ and Colin gets $\frac{-11}{3}$.

d)

$\begin{bmatrix}\frac{2}{3} & \frac{1}{3}\end{bmatrix} \cdot \begin{bmatrix}3 & 4 & 2\\ 5 & -2 & 7\end{bmatrix} \cdot \begin{bmatrix}\frac{1}{5}\\ \frac{2}{5}\\ \frac{2}{5}\end{bmatrix}$

$= \begin{bmatrix}\frac{6}{3}+\frac{5}{3} & \frac{8}{3}-\frac{2}{3} & \frac{4}{3}+\frac{7}{3}\end{bmatrix} \cdot \begin{bmatrix}\frac{1}{5}\\ \frac{2}{5}\\ \frac{2}{5}\end{bmatrix}$

$= \begin{bmatrix}\frac{11}{3} & \frac{6}{3} & \frac{11}{3}\end{bmatrix} \cdot \begin{bmatrix}\frac{1}{5}\\ \frac{2}{5}\\ \frac{2}{5}\end{bmatrix} = 3.$

It means that Rose gets 3 and Colin gets -3.

5

$$\begin{bmatrix}0 & 7/12 & 5/12 & 0\end{bmatrix} \begin{bmatrix}0 & 2 & -3 & 0\\ -2 & 0 & 0 & 3\\ 3 & 0 & 0 & -4\\ 0 & -3 & 4 & 0\end{bmatrix} = \begin{bmatrix}1/12 & 0 & 0 & 1/12\end{bmatrix}$$

Thus Row's guarantee is zero with this strategy. Transpose and negate to get Col's results as the game is symmetrical.
As the guarantees for both players with this strategy are equal, this is a Von Neumann solution.

6

a)

$$\begin{bmatrix}\frac{1}{4} & \frac{1}{4} & \frac{1}{2}\end{bmatrix} \cdot \begin{bmatrix}2 & -1 & 8 & 3\\ 1 & 3 & 1 & 2\\ 5 & 2 & -1 & 1\end{bmatrix} = \begin{bmatrix}\frac{13}{4} & \frac{6}{4} & \frac{7}{4} & \frac{7}{4}\end{bmatrix}$$

Then, Rose has a guarantee of $\frac{6}{4} = \frac{3}{2}$.

b)

$$\begin{bmatrix}\frac{1}{5} & \frac{3}{5} & \frac{1}{5}\end{bmatrix} \cdot \begin{bmatrix}2 & -1 & 8 & 3\\ 1 & 3 & 1 & 2\\ 5 & 2 & -1 & 1\end{bmatrix} = \begin{bmatrix}\frac{10}{5} & \frac{10}{5} & \frac{10}{5} & \frac{10}{5}\end{bmatrix}$$

And then, Rose is guaranteed $\frac{10}{5} = 2$.

c)

$$\begin{bmatrix}2 & -1 & 8 & 3\\ 1 & 3 & 1 & 2\\ 5 & 2 & -1 & 1\end{bmatrix} \cdot \begin{bmatrix}\frac{1}{4}\\ \frac{1}{2}\\ \frac{1}{4}\\ 0\end{bmatrix} = \begin{bmatrix}\frac{8}{4}\\ \frac{8}{4}\\ \frac{8}{4}\end{bmatrix}$$

Then, Colin has a guarantee of -2.

d)

$$\begin{bmatrix}2 & -1 & 8 & 3\\ 1 & 3 & 1 & 2\\ 5 & 2 & -1 & 1\end{bmatrix} \cdot \begin{bmatrix}0\\ \frac{1}{3}\\ \frac{1}{3}\\ \frac{1}{3}\end{bmatrix} = \begin{bmatrix}\frac{10}{3}\\ \frac{6}{3}\\ \frac{2}{3}\end{bmatrix}$$

Then, Colin has a guarantee of $-\frac{10}{3}$.
At last, note how strategies b and c both return the guarantee value of 2, and hence are the von neuman solution.

7

	Rock	Paper	Scissors	Lizard	Spock
Rock	0	-1	1	1	-1
Paper	1	0	-1	-1	1
Scissors	-1	1	0	1	-1
Lizard	-1	1	-1	0	1
Spock	1	-1	1	-1	0

A solution to this game is to play uniform randomly, which has a guarantee of zero for each player.

8

$$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} . \begin{bmatrix} 1 & 3 & 1 \\ 1 & -1 & 0 \\ 2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 + 2x_3 & 3x_1 - x_2 + x_3 & x_1 + 2x_3 \end{bmatrix}$$

Then the linear program looks like:

$$\begin{aligned} Max \quad & L \\ s.t \quad & L \leq x_1 + x_2 + 2x_3 \\ & L \leq 3x_1 - x_2 + x_3 \\ & L \leq x_1 + 2x_3 \\ & x_1 + x_2 + x_3 = 1 \\ & 0 \leq x_1, x_2, x_3 \end{aligned}$$

For which SageMath gives the solution: $[L, x_1, x_2, x_3] = [\frac{5}{3}, \frac{1}{3}, 0, \frac{2}{3}]$. Then, the strategy is $[\frac{1}{3}, 0, \frac{2}{3}]$ and the value is $\frac{5}{3}$.

9

Guarantee: $\frac{5}{3}$
Strategy: $[\frac{2}{3}, \frac{1}{3}, 0]$ or $[0, \frac{1}{3}, \frac{2}{3}]$
This verifies the correctness of our answer in 8. as the guarantee for C is equal to the guarantee of R, making this a Von Neumann solution.