first name **last name** **id#**

no devices **50 min** **2020 cgt quiz 1**

1. For nim{2,2}, draw the tree showing all possible continuations of the game.

2. At left are the sizes (in binary) of a 4-pile nim position. At right, for each pile: if there is a winning move then **give the new pile size**; if there is no winning move then **cross out that new position**.

	position	new position	new position	new position	new position
size	1 1 0 0		1 1 0 0	1 1 0 0	1 1 0 0
size	1 1 0 1 0	1 1 0 1 0		1 1 0 1 0	1 1 0 1 0
size	1 0 1 1 0 1	1 0 1 1 0 1	1 0 1 1 0 1		1 0 1 1 0 1
size	1 1 1 1 0 1	1 1 1 1 0 1	1 1 1 1 0 1	1 1 1 1 0 1	

3. Claim: a 4-pile nim position has at most 3 winning moves.

   Proof. Let P be a 4-pile nim position. If P has nim-sum zero then P has no winning move(s) and we are done, so assume P has positive nim-sum. Let c be the _____ column of P whose sum is odd. Let r be any row of P such that the entry in that row and column is _____.

   From P, a move is winning if and only if the entry in the binary sizes array at row r and column c is changed to 0, and the entries in the rest of that row are changed so that the nimsum of the new pile sizes is zero. There can be at most 3 such entries, because **(finish the proof)**

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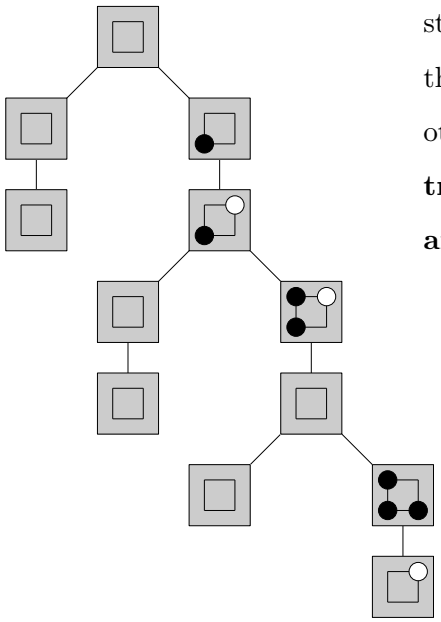
4. Claim: for any $n \geq 1$, nim$\{n, n\}$ is losing.

Complete the following proof by induction. Do not use Bouton's theorem.

Proof. Let t be any fixed positive integer. Assume that the claim holds for all $n < t$. We can assume that the first player moves to position $\{s, t\}$ where $0 \leq s < t$. Then the second player can move to position _____ and so the _____ player can win because **(now finish the proof)**

5. 

For go, the *net score* is black stones+territory minus white stones+territory. Here is part of a white strategy tree that shows that the net score is at most 1. (The root position is the empty board. Each other empty board represents pass.) **Draw the rest of the strategy tree** ... (I have pruned history-isomorphic children: do not add them) ... **and label each terminal position with its net score.**