

1. Recall Conway's definition of game sum: $G + H = \{\mathcal{G}^L + H, G + \mathcal{H}^L \mid \mathcal{G}^R + H, G + \mathcal{H}^R\}$.

In this notation, what does $\mathcal{G}^L + H$ mean? Answer in your own words. Also answer by assuming that $\mathcal{G}^L = \{G_1, \dots, G_t\}$, and then write the answer as a set.

2. Recall that game 0 is $\{ \mid \}$, game 1 is $\{0 \mid \}$. Using the definition of $-G$, show that the negative of game 1 is game -1 , i.e. $\{ \mid 0\}$.
3. For the game $G = \{\mathcal{G}^L \mid \mathcal{G}^R\}$, recall that $-G$ is defined as $\{-\mathcal{G}^R \mid -\mathcal{G}^L\}$. In this notation, assuming $\mathcal{G}^L = \{G_1, \dots, G_t\}$, what does $-\mathcal{G}^L$ mean. Answer in your own words, and also by giving the answer as a set.
4. Using the definition of $-G$, prove that $-(-G) = G$.
5. Draw the game tree for $G = \{0 \mid 0, *\}$ and find a clobber game with this tree (hint: try a line of 4 stones). Recall that $*$ is the game $\{0 \mid 0\}$. Explain the difference between game $*$ and game 0. (What are their game trees? What outcome class is each is?)
6. In class, we saw that any game that is a P-position behaves like 0 in sum: if Z is a P-position and X is any game, then the outcome class of $X + Z$ is the same as the outcome class of X . Prove this result.
7. Prove that the game $1 = \{0 \mid \}$ not not behave like zero: find a game V such that V and $V + 1$ are in different outcome classes.
8. Let W be any game that is not a P-position (so it must be an L-position or an R-position or an N-position). Prove that W does not behave like 0: find a game V such that V and $V + W$ are in different outcome classes.
9. Let A be the clobber game xxo . Let B be the clobber game xo .

For each game, either explain why it is a P-position, or draw its game tree:

$A, B, -A, -B, A + B, A + A, B + B, A - B, B - A$.