1. Review these definitions from CMPUT 272: isomorphism, bijection.

2. Let P_1, P_2 be these two tic-tac-toe positions.

```
     a  b  c          a  b  c
  1  X  .  .          .  .  X
  2  .  .  .          .  .  .
  3  .  .  .          .  .  .
```

   Here is an isomorphism between P_1 and P_2.

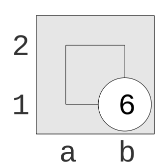
```
(a1 a2 a3 b1 b2 b3 c1 c2 c3)
(c1 c2 c3 b1 b2 b3 a1 a2 a3)
```

   Here, give another.

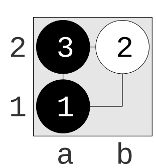
```
(a1 a2 a3 b1 b2 b3 c1 c2 c3)
(                          )
```

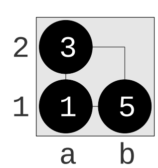
3. Consider a state S with position P and player-to-move X. Let Y be the opponent of X. Let Q_1 and Q_2 be two different but isomorphic positions that X can move to, and let T_1 and T_2 be the game tree rooted at (Q_1, Y) and (Q_2, Y) respectively. Describe the bijection $f : T_1 \to T_2$ that maps nodes in T_1 to nodes in T_2. Explain why, when we are analyzing S (say we want to find who wins, or a winning strategy, or the minimax score), we can prune T_2 from the game tree.

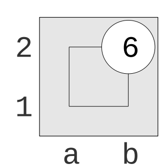
4. Here is a go position in a game that started from the empty board. Give a move history for this game, such that now 7.B[b2] is legal but 7.B[a1] is not.



5. i) The figures below show the 6-move history of a 2x2 go state. After move 3, how many move options did White have? Which move did White make? How do you know?



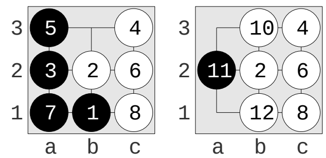




   ii) Let S be this go game after move 6, so with Black to play next. Give an isomorphism between the positions after 7.W[a2] and 7.W[b1] respectively. In the game tree, call the two corresponding nodes T_{7a2} and T_{7b1}.

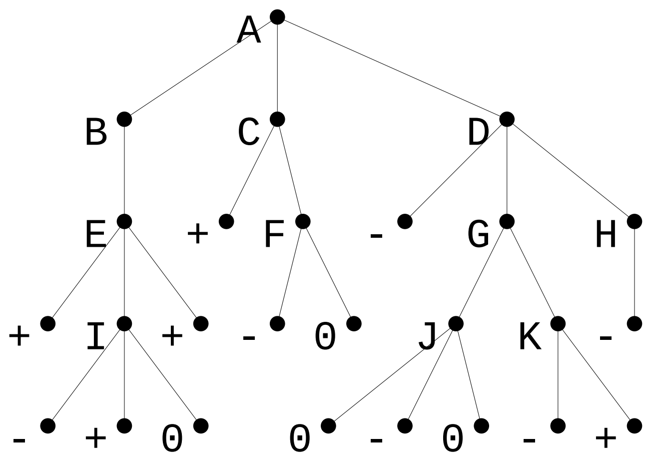
   iii) Assume that you want to find Black's best (minimax) strategy for S. Explain why you **should not** immediately prune one of T_{7a2}, T_{7b1} from the game tree.

6. Here is a go state. Net score is black stones+territory minus white stones+territory. i) If Black plays 13.pass, what is White's best reply? What will the final net score be? ii) If Black plays 13.a3, what is White's best reply? What will the final net score be?



7. Here is a win-loss-draw game tree. Leaf scores are for the 1st player. i) For this game, draw a tree that is a minimax strategy for the 1st player. ii) For this game, draw a tree that is a minimax strategy for the 2nd player. iii) What is the 1st-player minimax value for this game? Explain briefly.

iv) What is the smallest number (possibly 0) of leaves whose scores you need to change so that the 1st-player minimax score is + (win)? − (loss)? 0 (draw)?



8. Let B be the opponent of player A. Recall: for a game state G, if player A follows a winning strategy, then every continuation of the game ends at a terminal state where A wins.

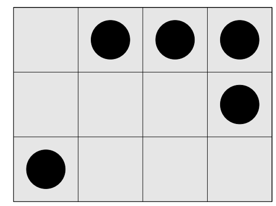
Notice: if A follows strategy (tree) S_A and B follows strategy (tree) S_B then the game ends at the unique leaf that is in both S_A and S_B.

Assume G is zero-sum, A has a winning strategy S_A, and B follows some strategy S_B. Explain why S_B cannot be a winning strategy.

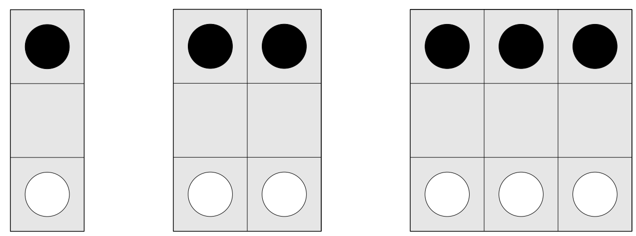
9. Recall: *A wins S* means that player A has a winning strategy for state S. Consider any win-loss combinatorial game G. Let G_1 (resp. G_2) L plays first (resp. second). Using Zermelo's theorem, prove the fundamental theorem of combinatorial game theory, namely that exactly one of these conditions holds:

- L wins G_1 and G_2,
- R wins G_1 and G_2,
- L wins G_1 and R wins G_2,
- R wins G_1 and L wins G_2.

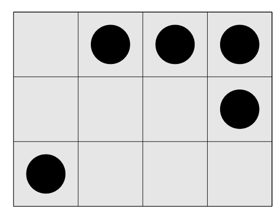
10. Here we represent a chomp position with a checkerboard:
    i) If we play chomp like this, give the rule for making a legal move.
    ii) Who wins this chomp position, N (next player) or P (previous player)?



11. Who wins each of these breakthrough games?



12. For the empty cells of this board, for domineering, is this a P-position, N-position, L-position, or R-position?



13. Consider the sum of domineering and chomp, each played on a 2x3 board. If Left plays first, who wins? if Right plays first, who wins? Prove your answers.

14. Consider the sum of 2x3 domineering and 3x3 breakthrough. If Left plays first, who wins? if Right plays first, who wins? Prove your answers.