cgt **homework 1** **do not hand in** **no marks**

1. For nim(2,2), draw the tree showing all possible continuations of the game, and then draw a tree showing a winning second-player strategy.

2. For each nim position, find all winning first moves. Show your work.

   (15,13,11,6,2) (5,7,9) (1,3,7,15,31) (31,31,31)

3. Claim: every move from a nim position whose nim-sum is zero leaves a position whose nim-sum is not zero. i) Prove the claim. ii) The claim is part of Bouton's theorem: state the other part.

4. Give a 4-pile nim position with exactly 2 winning moves. Justify briefly.

5. Prove/disprove: a k-pile nim position has at most k winning moves.

6. For $a \geq 1$, without using Bouton's theorem, prove by induction that the second player wins nim(a, a).

7. For Go, explain (i) the two kinds of legal move (ii) the game-termination condition (iii) how the game is scored.

8. In nim, a state is a position and the player-to-move. In go, a state is a position and a move history. Explain why move history is needed in a go state but not a nim state.

9. Section 2x2 in `https://webdocs.cs.ualberta.ca/~hayward/396/ssgo.pdf` shows a black strategy (with a tree) with which Black wins by at least 1 (against all possible White strategies). By drawing a tree, give a white (2nd-player) strategy for 2x2 Go that shows that white loses by at most 1 (against all possible Black strategies).

10. For a go position and a color, a *group* is a maximal connected set of stones of that color. Answer these questions for this position, from Figure 3.1 in *Mathematical Go: Chilling Gets the Last Point* by Berlekamp and Wolfe. (i) Give the number of black groups, white groups, black stones, white stones, black territory, white territory and the final score. (ii) Assume White now makes a non-pass move, and then both players pass. What is White's best move? Explain briefly. (iii) Repeat (ii) for Black. (iv) From the position, assume White makes some number non-pass moves and that Black passes after each and then White passes. What is the best score that White can achieve? Explain briefly. (v) Repeat (iv) with colors exchanged (Black makes non-pass moves).