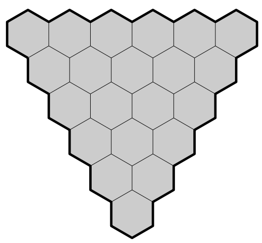
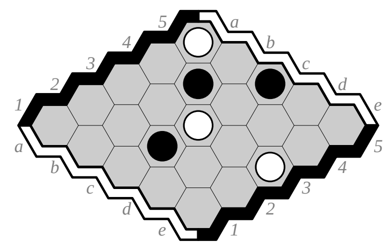
asn. 1 cmput670 gamesgraphsalgms ... due 1050am thu jan 23

Work alone. Acknowledge all resources, including discussions, texts, urls, etc. Non-detailed discussion with a non-group member is allowed, but must be summarized and acknowledged by all involved. Viewing or exchanging written work, even in rough or preliminary form, is not allowed.

1. Acknowledge all resources. Acknowledge that you have read and understood the UAlberta document Understanding Plagiarism `http://www.science.ualberta.ca/en/FacultyAndStaff/~/media/science/Faculty%20And%20Staff/Documents/Understanding_Plagiarism.ashx`.

2. Find all winning opening moves on the Y board below. Prove your answer is correct.





3. In a 2-player no-draw game such as Hex, for a state X and a player P with opponent Q, a strategy S is *best* for (X, P) if either (i) P wins with S, and S minimizes the number of moves that Q can force P to play to reach a win, or (ii) P has no winning strategy from X, but S maximizes the number of moves Q has to play to reach a win. A move is *best* if it is the first move in a best strategy.

   For the above Hex position, with black to play find all winning moves, and find all best moves. Repeat for white to play. Prove your answers are correct.

4. For Hex, Beck proved that opening in the acute corner loses. Does this proof also hold for Y? Justify carefully.

5. For a positive integer k, a k-Y board is a Y board with k cells on each side. Orient Y boards so that the top is flat. Label cells by row (counting from the top side) and column (counting from the left side). E.g., cell (2,3) is in row 2 and column 3.

   (i) In Schensted's Y reduction, give the 3-cells of the k-Y board that correspond to cell (x, y) of the $(k-1)$-Y board.

   (ii) Let α be a set of two adjacent cells on the $(k-1)$-Y board. Prove that β, the corresponding set of cells on the k-Y board, is connected.

6. Many cell-coloring games such as Hex and Y are *monotone*: if a player Z can win from a position X, then Z can win from the position X' obtained by adding one or more Z-stones to X. Prove that Y is monotone. Hint: argue by induction on the number of empty cells in X.