2. (i) There are many Black weak winsets.

   (ii) The intersection of the two sets from (i).

   (iii) c2 is the only winning move. You could show this by picking a small (maybe 4?) number of Black weak winsets, including ones with key c2. For example, black wins by c2 with c1d1 at top, so any white move into a1 b1 a2 b2 a3 loses. black also wins by c2 with either of c1,d1 and also a1 b1 a2 b2 a3 at top, so white move to c1, or d1 loses. Also, black has a winset with by playing a5 on the bottom so any white move to the bottom right loses; similarly black wins by playing e4 on the bottom right, so any white move to the bottom left loses. so the only winning move is c2.

   (iv) The best estimate is based on White's shortest win against all possible Black strategies. After 7.Wc2, Black can play so that White wins only on move 19. This will be discovered by the solver at tree depth corresponding to move 19, where the root of the tree corresponds to move 6.

   If you use a tree, the total number of nodes will be about 2e14 before you find that White can win by move 19. See below.

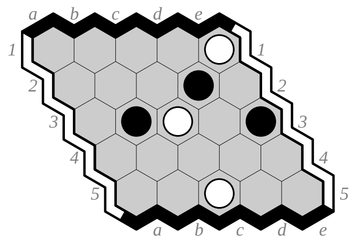
```
6  1  1
7  19  20
8  342  362
9  5814  6176
10  93024  99200
11  1395360  1494560
12  19535040  21029600
13  253955520  274985120
14  3047466240  3322451360
15  33522128640  36844580000
16  335221286400  372065866400
17  3016991577600  3389057444000
18  24135932620800  27524990064800
19  168951528345600  196476518410400
```

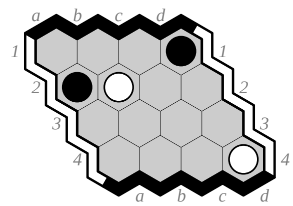
   If you use a dag, the number of states is astronomically smaller, less than 2e8.

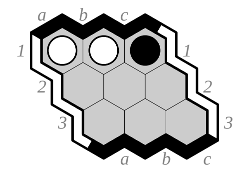
```
6  1
7  19
8  342
9  2907
10  23256
11  116280
12  542640
13  1763580
14  5290740
```

```
15 11639628
16 23279256
17 34918884
18 46558512
19 46558512 170694557
```

(v) (a) Ba1 kills b1, Bb1 kills a1. (b) Any Black winset containing c1 must also contain b1 or c2, in each case c1 is not needed. (c) one argument is that c4 is not in the white mustplay, so it is inferior to everything else







3. (i,ii) There are 6 choose 3 = 20 3-element subsets of the remaining 6 cells. All but 7 of them yield a Black win. So Black wins with probability 13/20 = .65.

   (iii) Assume Black plays next. Then Black wins with probability 1, e.g. by playing b3, and then on the next move b2, and if that is occupied c2.

   (iv) Assume White plays next. The White strategy b2, then the pair a3,b2 with probability .5 and the pair c2,c2 with probability .5 wins with probability at least .5 against any possible black strategy.

4. Consider the MCTS tree under the heading `Backpropation` near the top of this page:

   `https://en.wikipedia.org/wiki/Monte_Carlo_tree_search`

   (The root node is labelled 11/22; white nodes indicate player to move; dark nodes indicate opponent to move; all scores on all nodes are for P, the root node player.)

   (i) Show the four trees (selection, expansion, simulation, backpropagation) for the next iteration of MCTS. Assume that selection always chooses the child with maximum win rate; if more than one child has max win rate, the child with fewest trials is picked; if more than one such child exists, one of these is picked at random. Assume that the simulation result is a loss for P.

   (ii) Show the tree after 10 more iterations have finished. Assume that the result of each simulation is a loss for P.

   (iii) Now modify the MCTS algorithm so that during expansion, 2 children are generated, and repeat (ii).

   (iv) Now modify the MCTS algorithm so that during expansion, 2 children are generated, and each is initialized with a value of 1 win and 1 trial.

5. Read (or skim) the survey of MCTS.

   `http://www.cameronius.com/cv/mcts-survey-master.pdf`

   If you had to do an exploratory project on MCTS, what (sub)section of this paper would you find most interesting as a starting point? Explain briefly (50-200 words).