3. White 13, Black 12 so White wins by 1.

   Black 11 was captured.

   From the webpage, perfect play for 5x5 Go after 1.B[c2] ends with Black winning by 3.

   The TT score of this position is White 5 stones + 2 points, Black 11 stones + 2 points, so Black wins by 6.

   But Black can capture all white stones, so Black can win by 25.

4. in a straight line (on the surface of the earth)

   it is a lower bound on the surface road distance, so the path found by A* will be a shortest path. it's also easily computed from longitude and latitude.

```
 A   C   D   F   L   M   O   P   R   S   T   Z
366 160 242 176 244 241 380 100 193 253 329 374

initialize:  (other costs initially infinite)
           O
cost       0
priority 366
* current O cost 0
---------------------------------

O nhbrs:
Z newcost 0+71
S newcost 0+151

       Z    S
Cost   71   151
heur   374  253
pri    445  404
            *

Current S cost 151
---------------------------------

S nbrs:
O done
A newcost 151+140 291 update
F newcost 151+99  250 update
R newcost 151+80  231 update
```

```
        Z    S    A    F    R
Cost   71   151  291  250  231
heur   374  253  366  176  193
pri    445  404  657  426  424
                           *


Current R cost 231
---------------------------------

R nhbrs:
S Already done
C newcost 231+146 377 update
P newcost 231+97  328 update


        Z    S    A    F    R    C    P
Cost   71   151  291  250  231  377  328
heur   374  253  366  176  193  160  100
pri    445  404  657  426  424  537  428
                                     *

Current P cost 328
---------------------------------

P nhbrs:
R Already done
C Already done
B newcost 328+101 429 update


        Z    S    A    F    R    C    P    B
Cost   71   151  291  250  231  377  328  429
heur   374  253  366  176  193  160  100   0
pri    445  404  657  426  424  537  428  429
                                          *
```

5. if the heuristic is 0, then the A* priority function will be just the min-distance-so-far to each node, so A* becomes Dijkstra's algorithm (except that it is bit slower, because of the extra useless additions).

6. Yes, we can always get from A to B. A is solvable, so we can get from the start to A, so we can get from A to the start (reverse the move sequence). B is solvable so we can get from the start to B. So combining these two sequences, we can get from A to the start and then to B.

   This question is asking for the number of connected components in the search space of 9! 3x3 positions. By the previous question, we know that the set of solvable positions forms one connected component. But is the set of unsolvable positions also connected, or does it break into more than one component?

   The set of unsolvable positions is also connected. Here is one proof. Take each unsolvable position, and change label 8 to 10. This changes the number of inversions by exactly one, because

   in the original position, a tile t in set$\{1,\ldots,7\}$ is inverted with 9 if and only if, in the relabelled position, it is out of order with tile 9 (because both t and 9 are in the same position in each position)

   in the original position, a tile t in set$\{1,\ldots,7\}$ is inverted with 8 if and only if, in the relabelled position, it is out of order with tile 10 (because t's position is unchanged, and 8 and 10 occupy the same position, and t is out of order with respect to 8 if and only if it is out of order with respect to 10).

   in the original position, 8 and 9 are inverted if and only if 10 and 9 are not inverted (because the positions have not change, but 8 is less than 9 whereas 10 is greater than 9).

   So this relabelling causes the number of inversions to change by exactly 1, so each unsolvable position because solvable. Now use the previous question: the sequence of moves will take A (relabelled) to B (relabelled). Now change the labels back: relabel 10 as 8. The same sequence takes original A to original B.

7. Here is an example from sliding left:

```
position      tile permutation
7 3 4 2       7 3 4 2 6 5 1
6   5 1

7 3 4 2       7 3 4 2 6 5 1
6 5   1
```

   Notice that a shift left or right does not change the row-by-row tile permutation, so the number of inversions does not change.

8. Here is an example from sliding up:

```
position      tile permutation
7 3 4 2 8     7 3 4 2 8 6 5 1 9
```

```
6   5 1 9

7   4 2 8    7   4 2 8 6  3 5 1 9
6 3 5 1 9
```

In a position with 5 columns, if tile t is shifted up, then in the tile permutation, tile t jumps left 5 positions, so changes places with exactly 4 other tiles, so the change in the number of inversions will be $\delta = \pm 1 + \pm 1 + \pm 1 + \pm 1$. Since $\pm 1 + \pm 1 = -2$ or 0 or 2, delta is in $\{0, \pm 2, \pm 4\}$.

9. yes. consider an arbitrary 3x3 position A. If A is solvable, then by the first part of question 6, we can get A to the solvable goal position below. If A is not solvable, then by the second part of question 6, we can get A to the unsolvable goal position below. In each case, we have 1,2,3 in their correct positions, as required.

```
solvable goal           unsolvable goal
1 2 3                   1 2 3
4 5 6                   4 5 6
7 8                     8 7
```

Here is the outline of a different proof: we can transform the starting position at left to each following position to its right.

```
* * *    1 * *    1 2 *    1 2 3    1 2 *    2   *    2 3      1 2 3
* * *    * * *    * * *    * * * or * * 3    1 * 3    1 * *    * * *
* *      * *      * *      * *      *   *    * * *    * * *    * *
```