2
basics

It is a happy talent to know how to play.
Ralph Waldo Emerson

Here are some basic tips for playing Hex. Skip this chapter if you are already an expert or would rather learn on your own.

Let's start small. In Figure 2.1, for B and for each board, can you find all winning moves and — for each such move — an accompanying strategy? A *winning move* is the first move of a winning strategy. A *winning strategy* wins against every possible opponent strategy. Answers start on the next page.

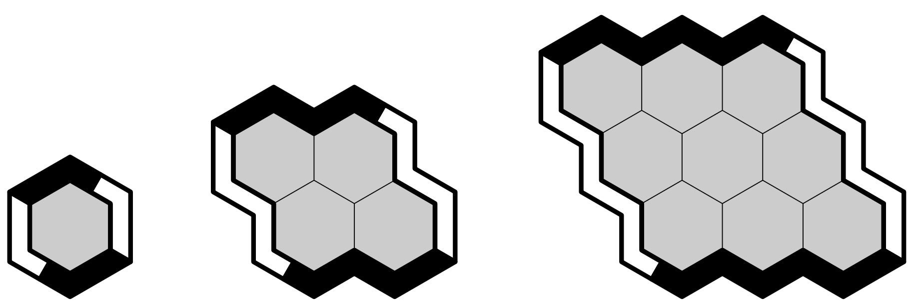

Figure 2.1. Can you find all winning B-moves?

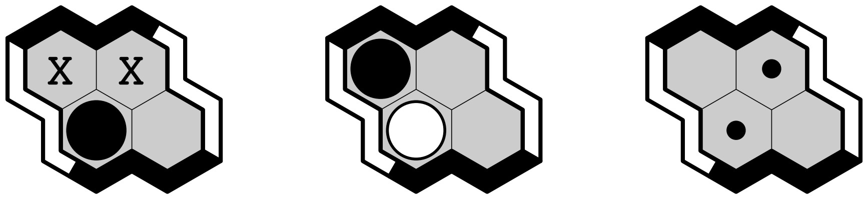

Figure 2.2. A winning B-move, a losing B-move with winning W-reply, and all winning B-moves.

Answers for 1×1 and 2×2 boards. On the 1×1 board the only move wins! In Figure 2.2, in the first diagram the B-move wins: if W plays at one X then B can reply at the other. We call this two-cell connection pattern a *bridge*. In the next diagram the B-move loses to the W-reply, since now W has a winning bridge. The other two opening moves are rotationally isomorphic to these two, so the last diagram shows all 2×2 winning B-moves.

Before answering for the 3×3 board, we need a bit of theory ...

2.1 block

Consider this advice from Piet Hein:

The point of Hex is to block your opponent. This is ... part of the essence of the game, that offense and defense blend together. A barrier against your opponent becomes a path for you. Think of Hex as two labyrinths in combat.

In Figure 2.3 where should W play? Answer on the next page.

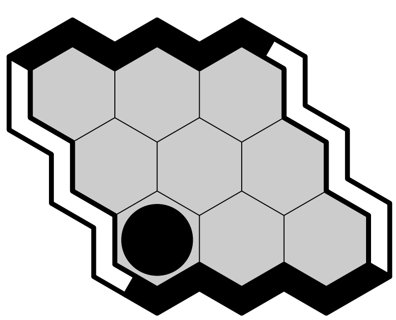

Figure 2.3. Where should W play?

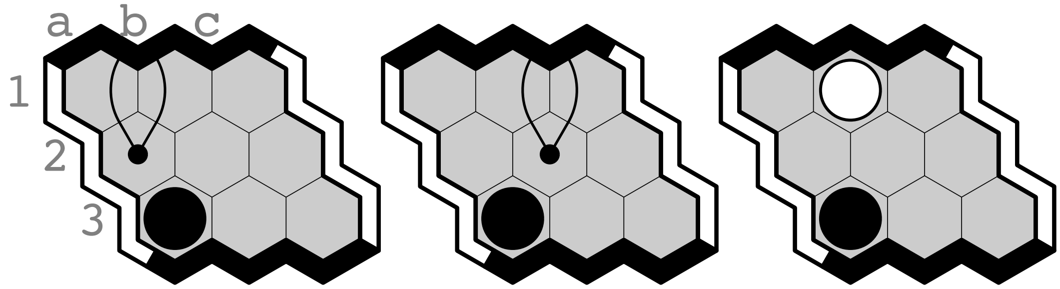

Figure 2.4. W should block the two B-threats.

Answer. In Figure 2.4 the first diagram (and the next) shows a three-cell B-wins threats: play at the dot and then bridge to the top. The last diagram shows where W should play: at the only cell that interferes with each threat.

2.2 pairing

W's move in Figure 2.4 merely delays B's march to victory. Figure 2.5 shows how B can win. After 2.W[b1] and 3.B[c1], B has two win-threats: 5.B[b2] wins immediately, 5.B[c2] wins via a bridge. W can stop only one threat. Like a bridge strategy, B's strategy here is a *pairing*: whenever your opponent plays in one cell of a pair, you reply at the other cell. After 1.B[a3] in Figure 2.4, B wins with pairing {{a2, c1}, {a1, b1}, {b2, c2}, {b3, c3}}. The X- and Y-cells in Figure 2.5 show part of this pairing. In the right diagram of that figure, can you find a B-wins pairing? Answer on the next page.

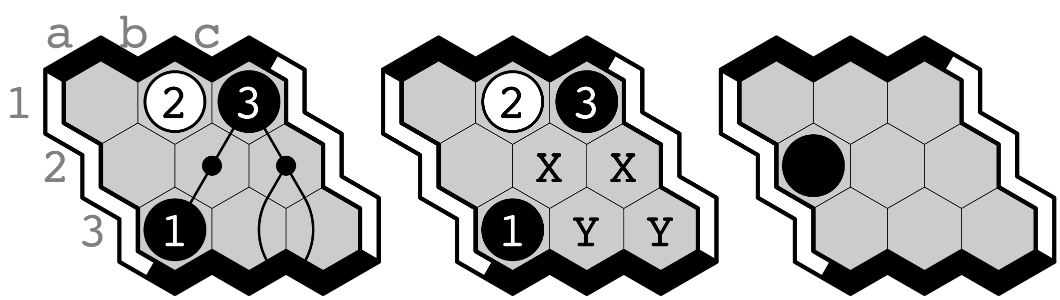

Figure 2.5. (left, middle) 3.B[c1] wins. (right) Can you find a B-wins pairing?

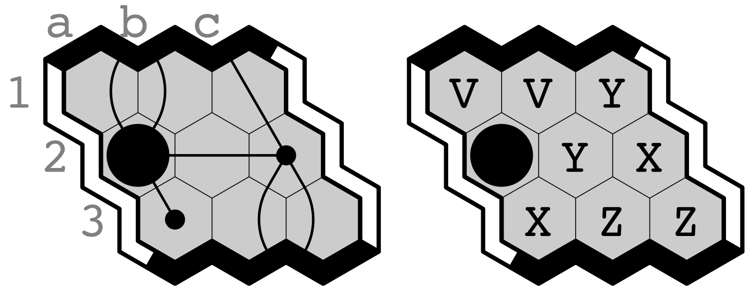

Figure 2.6. A B-wins pairing strategy.

Answer. Figure 2.6 shows how B wins in the bottom of Figure 2.5. Use the {a1,b1} bridge to join a2 to the top. Now B has two threats, shown by dots: a3, or c2 plus bridges {b2,c1} and {b3,c3}. This is a pairing strategy, as shown in the right diagram.

2.3 3×3 openings

Using what you have learned — bridges, blocking, pairs — can you find all winning first moves for the 3×3 board? Answer below.

Answer. We saw in Figures 2.4 and 2.5 that opening in the obtuse corner wins. We saw in Figure 2.6 that opening off-center on the opponent's side wins. We see in Figure 2.7 (left) that opening in the center wins. So five opening moves win. We see in Figure 2.7 (middle) that the other four openings lose. So we have found all 3×3 winning opening moves (right).

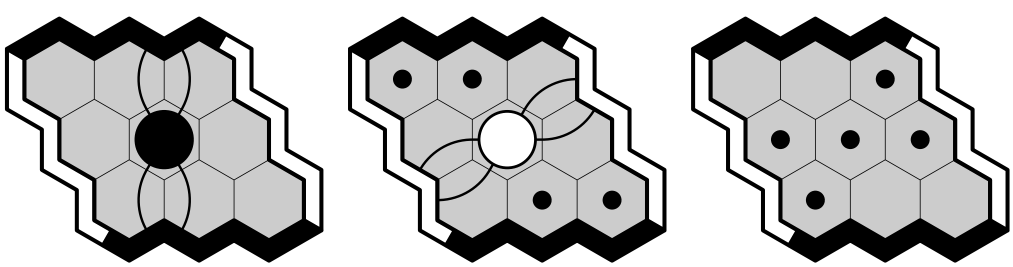

Figure 2.7. Center opening wins, four B-losing openings, all B-winning openings.

2.4 safe connection

The winning strategy in Figure 2.6 consists of several *safe connections*, namely point-to-point substrategies in which a player can join the two points even if the opponent moves next.

The bridge is the simplest safe connection, using only two empty cells. Figure 2.8 shows three connections that each join a stone group to a side of the board. We call them the 2X-2-X (starting from the side of the board, the first row 2 empty cells and a stone (X), the next row has 2 empty cells, and the next row has a stone), the 3-3-XX (3 empty cells, 3 empty cells, 2 stones) and the 4-3-1X (4 empty cells, 3 empty cells, 1 empty cell and a stone).

For each, can you find a strategy that allows B to join her stone group to the side, even if W moves next? Answer on the next page.

Fun fact: safe connections — also called virtual connections — also feature in the game of Go, where bridges are called *miai* (pronounced mee-eye), meaning *equal options*.

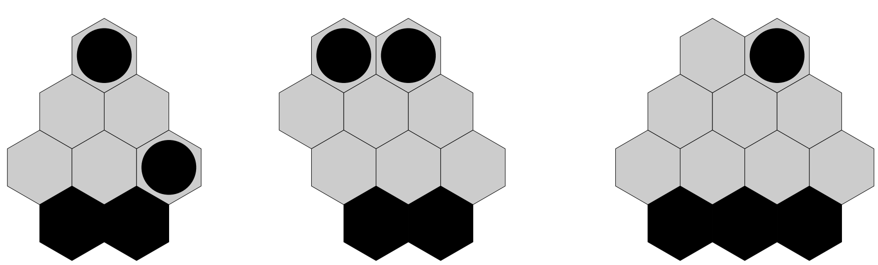

Figure 2.8. 2X-2-X (left), 3-3-XX (middle) and 4-3-1X (right) side connections. For each, can you find a connection strategy?

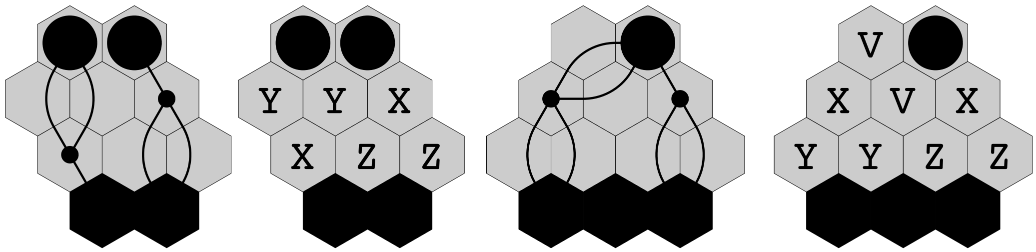

Figure 2.9. 3-3-XX and 4-3-1X strategies.

Answer. We leave the 2X-2-X strategy for you to find. Using the 3-3-XX and 4-3-1X strategies in Figure 2.9, B safely joins her stone group to the side, even if W plays first in the connection region.

2.5 6×6 puzzle

Let's apply what we have learned to a 6×6 puzzle. In Figure 2.10, where should B play? Remember Hein's advice: block the opponent. A *winning path* for a player is a set of empty cells that joins her two sides. One way to block the opponent is to play at a cell on one of her shortest winning paths. But some such cells block better than others.

The figure gives you a hint: we have shaded each cell that is on a shortest W-wins path. If B's next move is *not* in such a cell, then W wins easily: use three bridges after playing any dot in Figure 2.11. But which of these nine shaded cells gives a best move for B? Read on ...

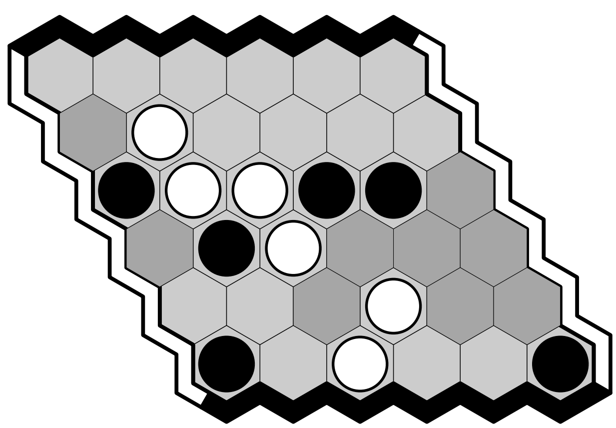

Figure 2.10. Where should B play?.

2.6 win-threats

To avoid defeat you must block each opponent win-threat. The shaded cells in the top diagrams of Figure 2.11 each show a W-win threat. B must block each threat, so play a cell that is shaded in both diagrams. There are five such cells, shown in the bottom diagram. Do any of these five moves win? Answer on next page.

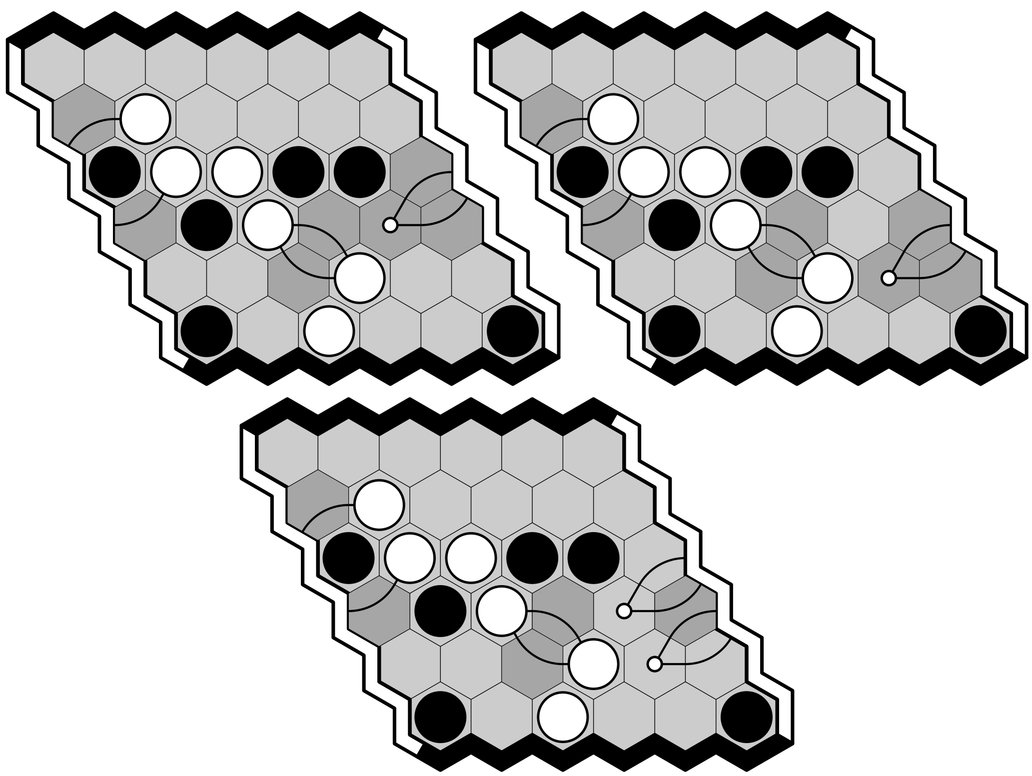

Figure 2.11. To interfere with each W-threat at top, B must take one of the five moves at bottom.

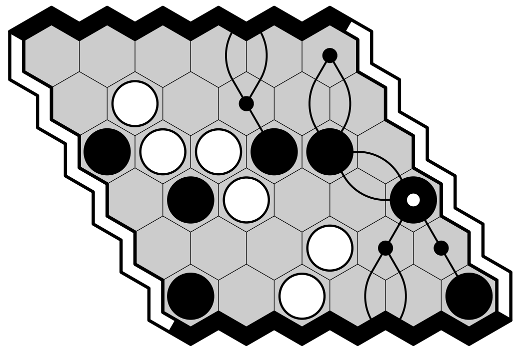

Figure 2.12. A winning B-move.

Answer. Here all five moves win. The four moves in the two W-bridges of Figure 2.11 win slowly: in each case W can restore the bridge but B can still win. The move in Figure 2.12 wins quickly: it joins the top with a bridge and a 3-3-XX connection and joins the bottom with a X2-2-X connection.

2.7 7-6-5-1X

Sometimes you cannot stop all opponent win-threats. In Figure 2.13 with W to play, who wins? Hint: see Figure 2.14. Answer below.

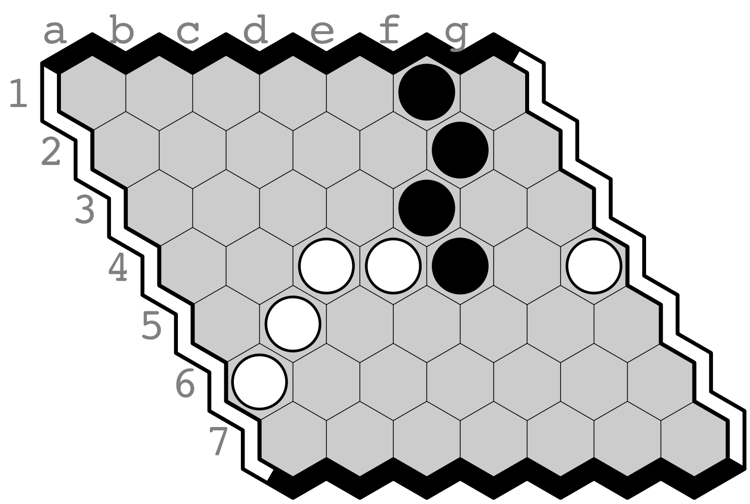

Figure 2.13. W to play: who wins?

Answer. B wins with a 7-6-5-1X side connection. If W plays any of {e5, e6, e7, f4, f5, f6, f7, g5, g6, g7} then B can reply at d5 as in the left diagram of Figure 2.14 and finish the win by maintaining the 4-3-X1 connection. Similarly, if W plays d7 then B can reply at e5. And if W plays any of {a7, b6, b7, c5, c6, c7, d5, d6} then B can reply at f5.

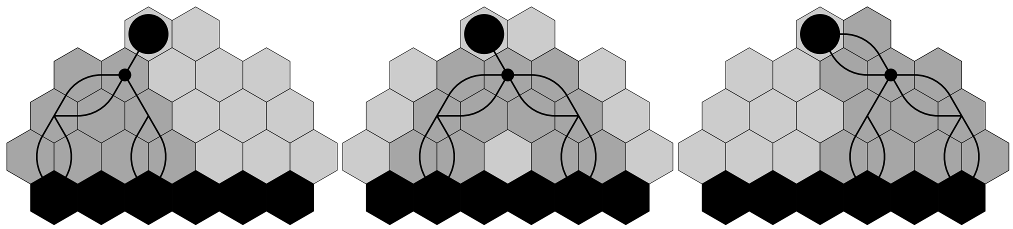

Figure 2.14. Three threats form the 7-6-5-1X.

2.8 weak, safe, super-strong connection

It pays to recognize different kinds of connection.

We have seen *safe connections* such as {a5, a6} in Figure 2.15, safely joining W's bottom side to b5, even if B plays next. And we have seen connection threats — also called *weak connections* — such as {e2, f1, f2} which (if W plays next at the dot) joins the W-stone d3 to the W-stone g1. To maintain a safe connection, a player can wait until the opponent attacks it before replying. To force a weak connection, a player must play there before the opponent.

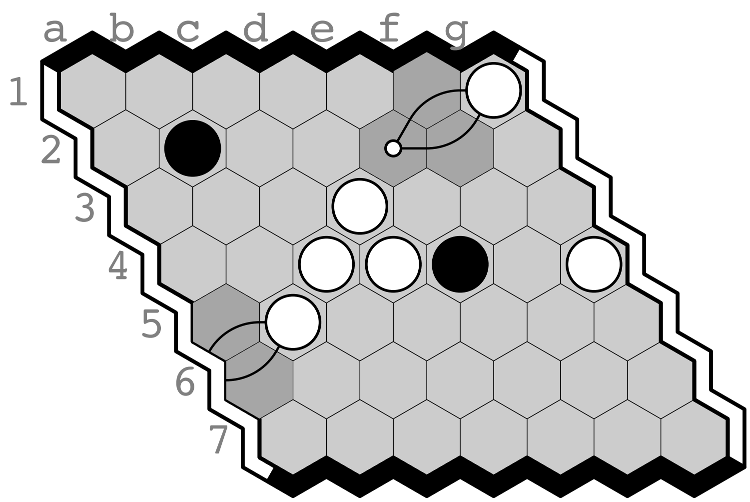

Figure 2.15. weak and safe connections

The shaded cells in Figure 2.16 form a *super-strong connection*. Here W can ignore a first B attack into these cells. Why? Because this connection is formed from *three* non-intersecting weak connections: {a5}, {a6}, and {b3,a3,a4,c3,b4}. B's initial attack misses two non-intersecting weak connections, which form a safe connection.

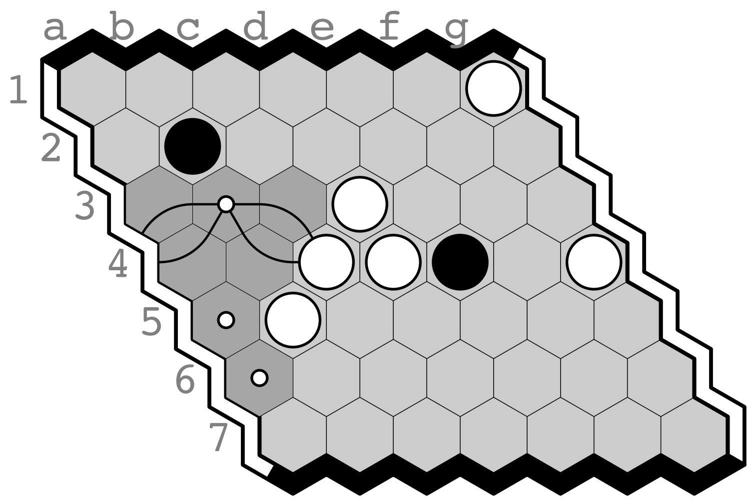

Figure 2.16. super-strong onnection

In Figure 2.16, where should B play? The shaded cells in Figure 2.15 form a W win-threat, so B must play at one of those five cells. But only one wins! Can you find it? Hint: it won't be either of the cells in the super-strong connection in Figure 2.16, so you have only three cells to check. Answer on the next page.

Answer. So far we know that the mustplay region is {e2, f1, f2}. The top diagram in Figure 2.17 shows another W win-threat. The intersection of this threat and our mustplay is {f2}: every other B-move loses. For example, if B plays a5 — in the superstrong W-connection in Figure 2.16 — then W can reply at e3 and win. So, every B-move except f2 loses. Does this B-move win?

Yes! The bottom diagram in Figure 2.17 shows that B has a safe connection after this move. This kind of safe connection is called a *ladder*, because the move pattern as the opponent tries to stop it looks like someone climbing a ladder. Ladders are common in Hex and — fun fact — also in Go.

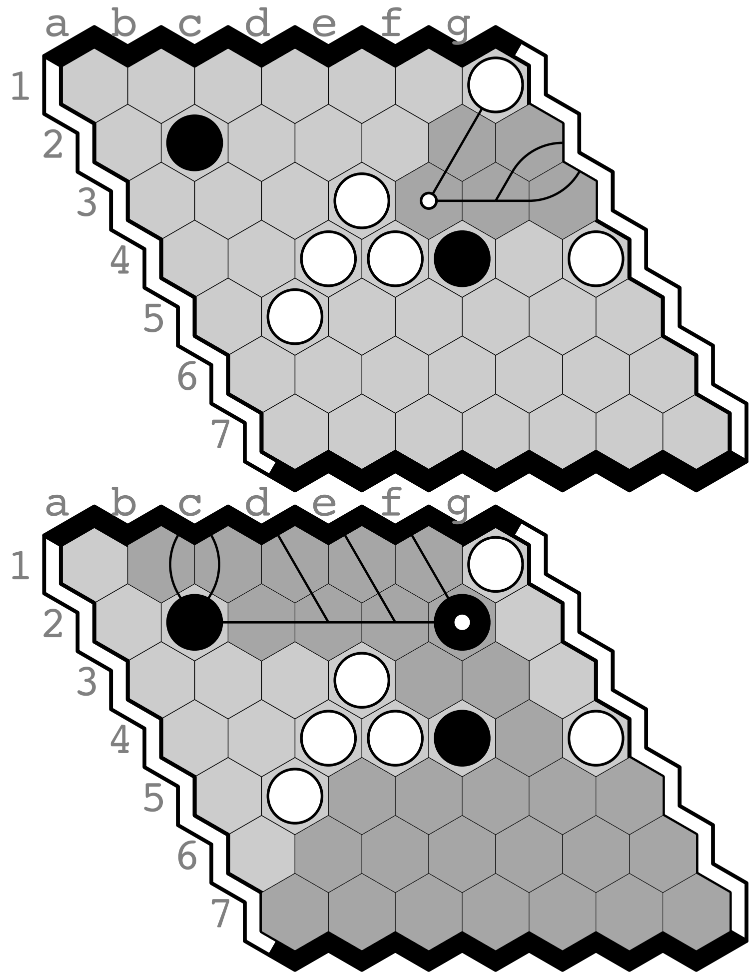

Figure 2.17. A W win-threat (top). B wins with a ladder, bridge, and 7-6-5-1X (bottom).

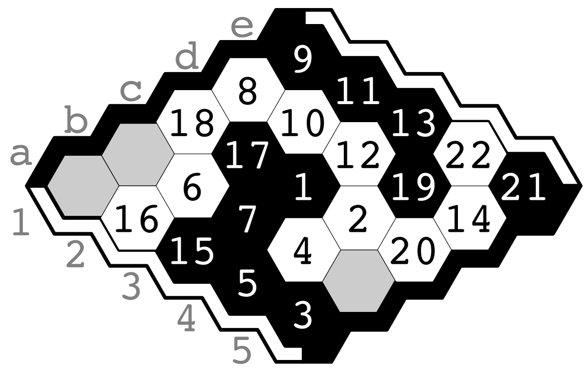

Figure 2.18. Can you find all blunders?

2.9 blunders

It can be fun — or painful! — to look back and analyze a game. Obviously, you would want to change any *blunder*, namely a losing move made when a winning move was available. How many blunders are in the game in Figure 2.18?

After 1.B[c3], B has a 4-3-X1 to the top and also the bottom, so move 1 wins and is not a blunder. So W has no winning move 2, so move 2 is not a blunder. Next we expect move 3.B[b4] — which restores the bottom 4-3-X1 after 2.W[c4] — instead of 3.B[a5]: is this a winning move? If not then it is the game's first blunder.

As shown in Figure 2.19, the game has exactly three blunders. For each, can you find a winning move? Answer on next page.

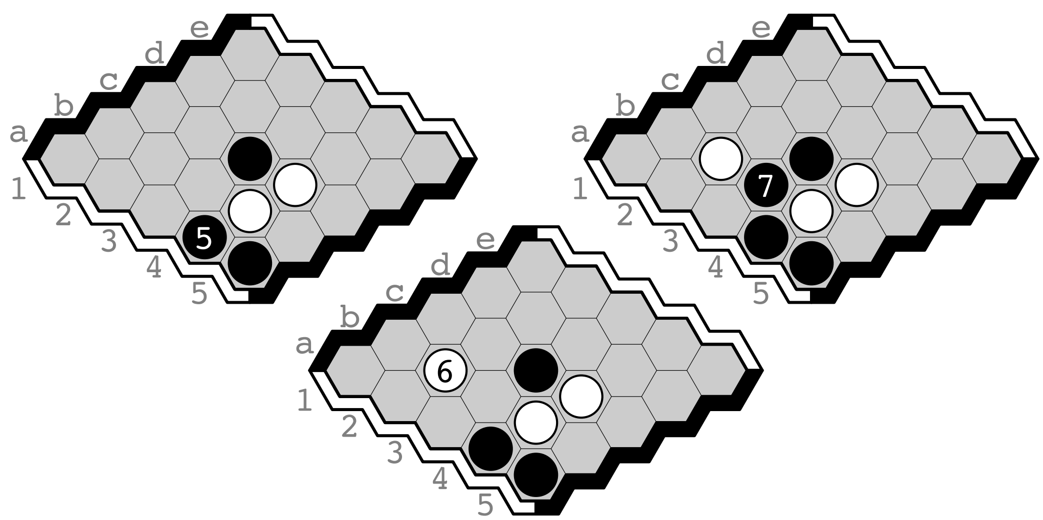

Figure 2.19. The game's only blunders. In each case, find a winning move. Answer on next page.

Answer. Each diagram of Figure 2.20 shows the only winning move for that position. For example, in the first diagram, B's only winning move is d3. We leave it to you to find corresponding winning strategies for these three winning moves, and to show that moves 5, 6 and 7 in Figure 2.18 are the game's only blunders.

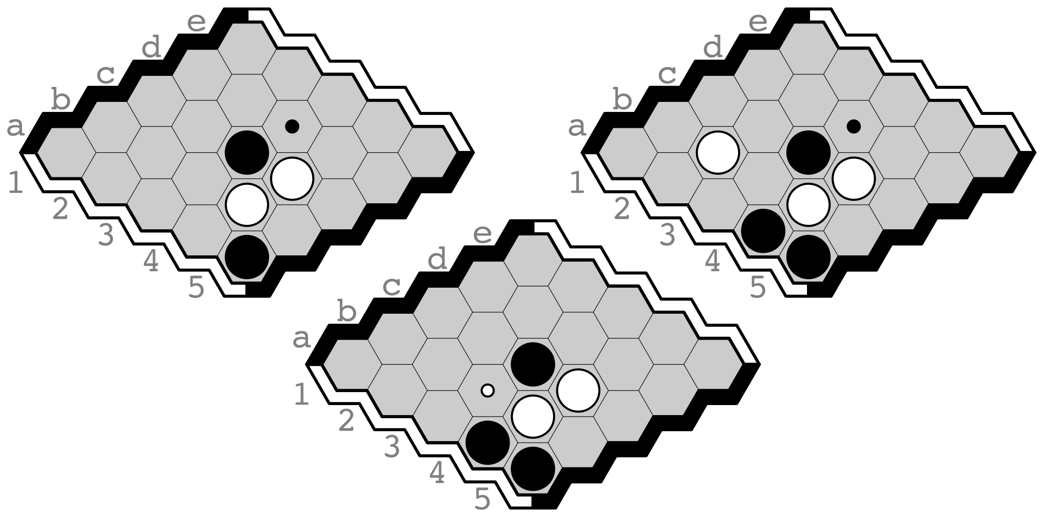

Figure 2.20. Each dot is the only winning move.

So now you can play Hex! If you want more practise, try to find all winning opening moves on the 4×4 and 5×5 boards. And try the puzzles in the next section.

2.10 puzzles

Try to find all winning moves.

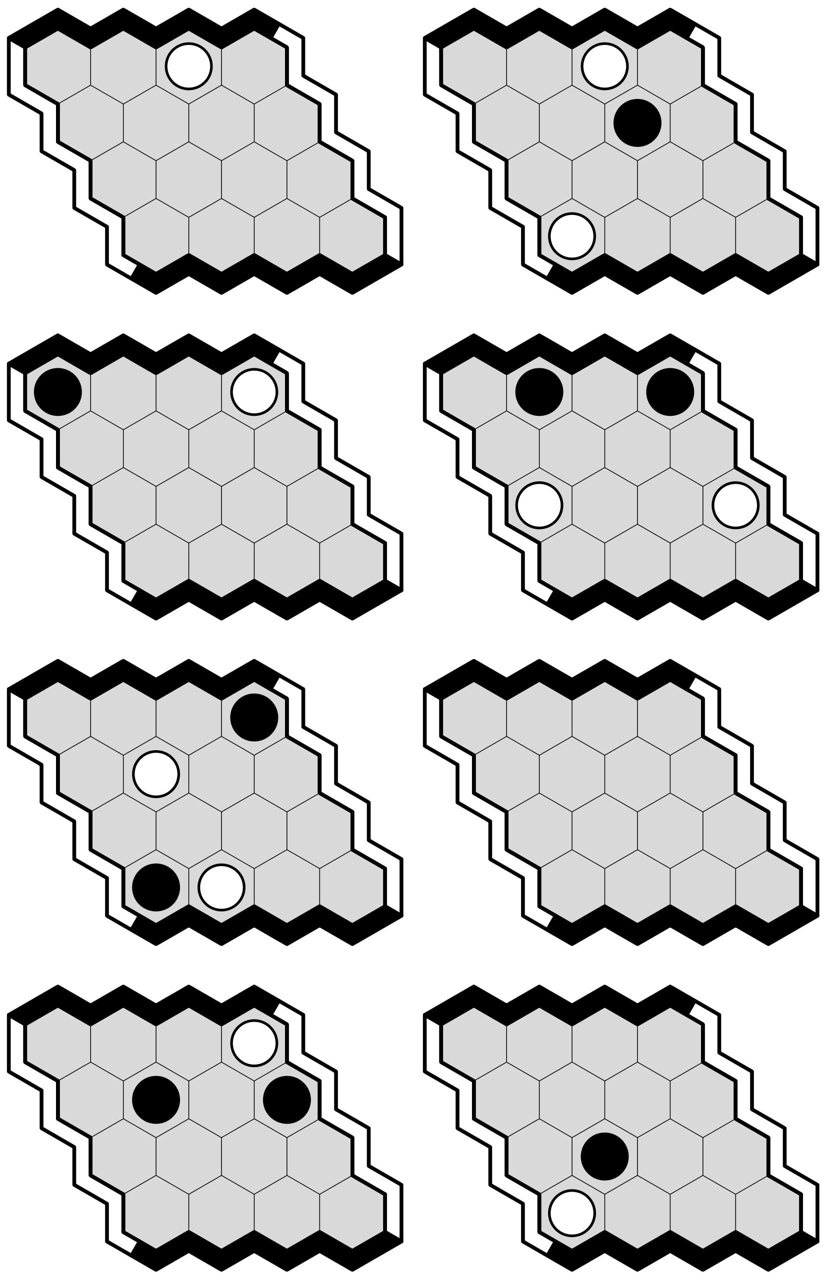

Figure 2.21. Top 3 rows: B to play. Bottom row: W to play.

2.11 solutions

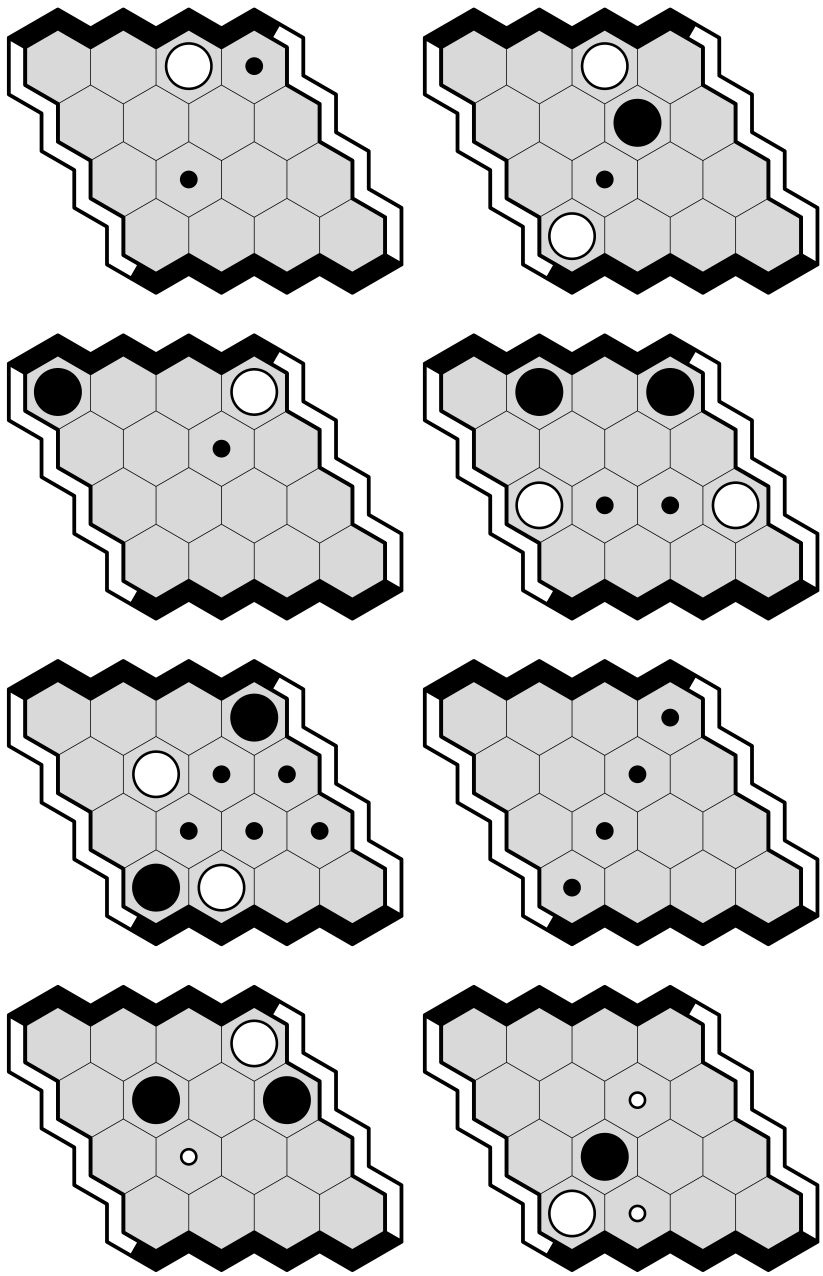

Figure 2.22. All winning moves.