cmput 355 2025 practice questions 6 (with answers)

Rose is the row player, Colin is the column player. RPS is rock-paper-scissors. Read handout `https://webdocs.cs.ualberta.ca/~hayward/355/rps.pdf` and answer these questions.

1. Give Rose's expected RPS payoff matrix.

Answer.

```
   R  P  S
R  0 -1  1
P  1  0 -1
S -1  1  0
```

2. This is modified RPS (MRPS): if Rose plays rock and Colin plays scissors, Rose wins 2, otherwise the game is the same as RPS. Give Rose's expected MRPS payoff matrix.

Answer.

```
   R  P  S
R  0 -1  2
P  1  0 -1
S -1  1  0
```

3. In MRPS, Colin follows strategy C_X: play R,P,S with resp. probabilities .5,.1,.4.

   a) Against C_X, how many possible strategies can Rose play?

   b) Give a strategy R_Y for Rose that is best against C_X.

   c) How can you answer b), given your answer to a)?

Answer.

a) An uncountable number. Rose can play RPS with resp. prob's (a, b, c) for every $\{a, b, c\}$ satisfying $0 \leq a, b, c \leq 1$, $a + b + c = 1$.

b) For $(a, b, c) = (1, 0, 0)$ Rose's expected payoff is .7. For $(a, b, c) = (0, 1, 0)$ Rose's expected payoff is .1. For $(a, b, c) = (0, 0, 1)$ Rose's expected payoff is $-.4$. So R_Y is $(a, b, c) = (1, 0, 0)$, play rock all the time.

c) A theorem tells us that Rose has a best strategy that is a single-action strategy, so it suffices to consider only the 3 single-action strategies and pick a best one.

4. Assume that Rose follows a strategy R_t with R,P,S resp. prob's a, b, c. If Colin plays column 1 against R_t, what is Rose's expected payoff? Repeat for columns 2 and 3.

Answer. $b - c$, $c - a$, $2a - b$.

5. Continuing from the previous question, how will Rose pick a, b, c to maximize her expected payoff?

Answer. For a fixed a, b, c Colin will pick the minimum of $\{b - c,\ c - a,\ 2a - b\}$. So Rose will pick a, b, c to maximize this minimum.

6. Continuing from the previous question, how many choices does Rose need to consider in order to find a strategy with best payoff?

Answer. A math theorem tells us that she only needs to consider those triples (a, b, c) that are the extreme points of the feasible region. You don't need to know what this means, but you should understand that it is good news, reducing the search space from an uncountable number to a number that is (worst-case) exponential in $a + b + c$.

7. Formulate Rose's problem as a linear program.

Answer.

```
max z   s.t.
    z <= b - c
    z <= c - a
    z <= 2a - b
    a,b,c >= 0
    a,b,c <= 1
    a+b+c=1
```

8. If you were working at home and had access to a computer and the internet, explain how you could find the solution to the previous question.

Answer. Use any LP-solver, e.g. sagemath's.

9. Formulate Colin's problem (find a best MRPS strategy) as a linear program.

Answer.

```
max z   s.t.
    z <= 2c - b
    z <= a - c
    z <= b - a
    a,b,c >= 0
    a,b,c <= 1
    a+b+c=1
```

10. Someone tells you that a Von Neumann equilibrium for MRPS is (3/12, 5/12, 4/12) for Rose, (4/12, 5/12, 3/12) for Colin, expected Rose payoff 1/12. Prove or disprove this.

Answer.

Proof. Against the Rose-strat, assume Colin plays column 1: then Rose's expected payoff is $5/12 - 4/12 = 1/12$. Similarly, you can check that Rose's expected payoff is exactly 1/12 when Colin plays column 2 and also column 3. So, against the Rose-strat, if Colin plays with probability (a, b, c) then Rose's expected payoff will be $a/12 + b/12 + c/12 = (a + b + c)/12 = 1/12$. So Rose can win at least 1/12.

Also, you can also confirm that if Colin plays his strategy, then Rose's expected payoff will be 1/12. So Colin can make sure that Rose never wins more than 1/12.

So 1/12 is the value for this game, and these two strategies and this value are indeed a VNE.

11. Someone tells you that a Von Neumann equilibrium for MRPS is (3/12, 5/12, 4/12) for Rose, (1/3, 1/3, 1/3) for Colin, expected Rose payoff 1/12. Prove or disprove this.

Answer.

Disproof. The Rose-strat and value are the same as in the previous question, so we know that by following this strategy Rose has expected payoff 1/12 against any Colin strategy.

However, if Colin follows his strategy and Rose plays rock, Rose's expected payoff is $2/3 - 1/3 = 1/3$, which is larger than 1/12, so Rose's expected payoff against Colin's strategy is at least 1/3. So these two strategies (Rose's and Colin's) give different expected payoffs for Rose, so this is not a VNE.

12. Rose and Colin play this game.

```
 1  0 -2
 3  5 -4
-6  7  8
```

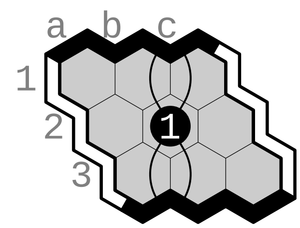
What is Rose's expected payoff if she plays $S = (.7, .1, .2)$ and Colin plays $T = (.4, 0, .6)$? (Express your answer as an arithmetic expression: you do not need to simplify.)

Answer.

```
(1st row) .7*.4* 1 + .7* 0* 0 + .7*.6* -2 +
(2nd row) .1*.4* 3 + .1* 0* 5 + .1*.6* -4 +
(3rd row) .2*.4*-6 + .2* 0* 7 + .2*.6*  8
```

Read handout `https://webdocs.cs.ualberta.ca/~hayward/355/andor.pdf` and answer these questions.

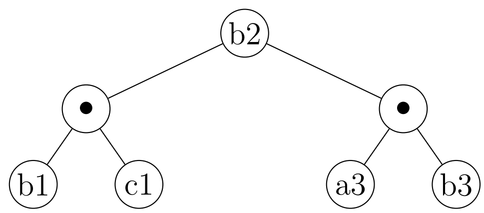
13. In each case, give this hex strategy in and-or form and draw the corresponding and-or tree:



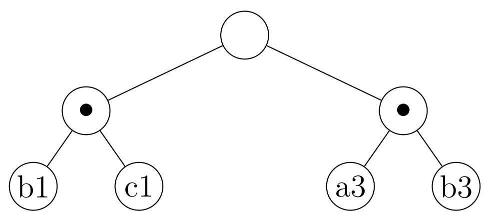
a) from the diagram, the 1st-player Black win-strat
b) after 1.B[b2], the remaining 2nd-player Black win-strat
c) after 1.B[b2] 2.W[b3], the remaining 1st-player Black win-strat

Answer.

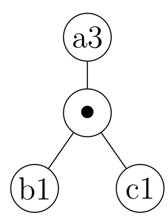
a) $b2 \wedge (b1 \vee c1) \wedge (a3 \vee b3)$



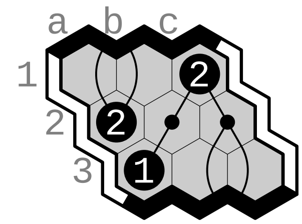
b) $(b1 \vee c1) \wedge (a3 \vee b3)$



c) $a3 \wedge (b1 \vee c1)$

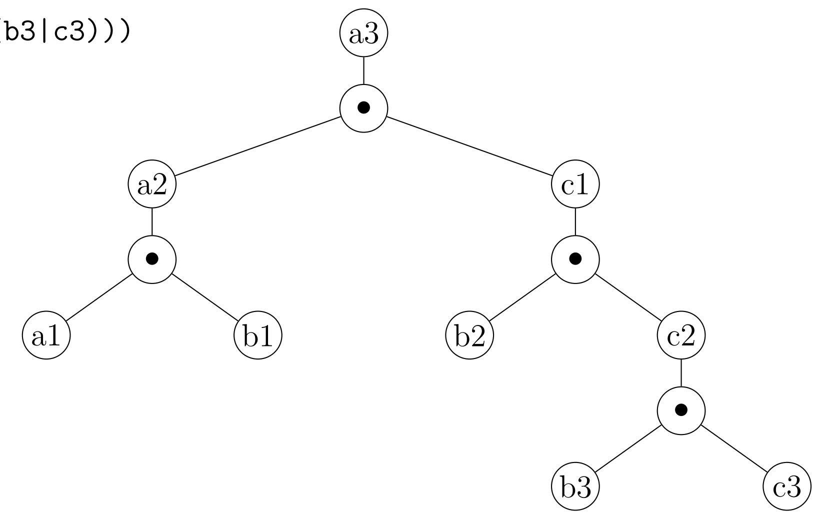


14. In each case, give this hex strategy in and-or form and draw the corresponding and-or tree:
a) from the diagram, the 1st-player Black win-strat
b) after 1.B[a3], the remaining 2nd-player Black win-strat
c) after 1.B[a3] 2.W[b1], the remaining 1st-player Black win-strat

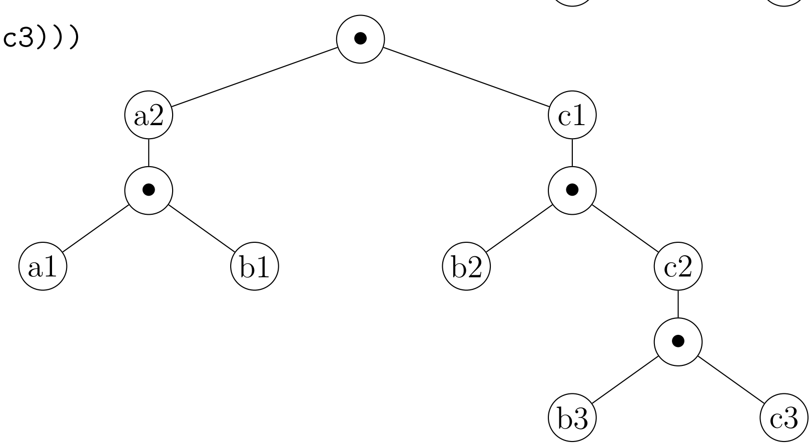


Answer.

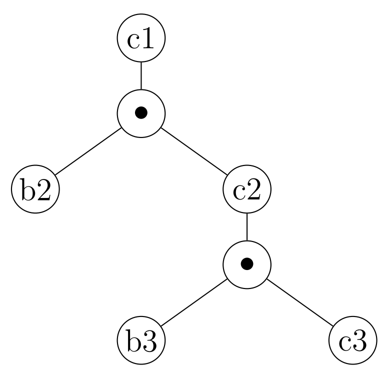
a) `a3. (a2.(a1|b1) | c1.(b2|c2.(b3|c3)))`



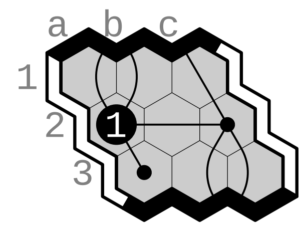
b) `(a2.(a1|b1) | c1.(b2| c2.(b3|c3)))`



c) `c1.(b2| c2.(b3|c3))`

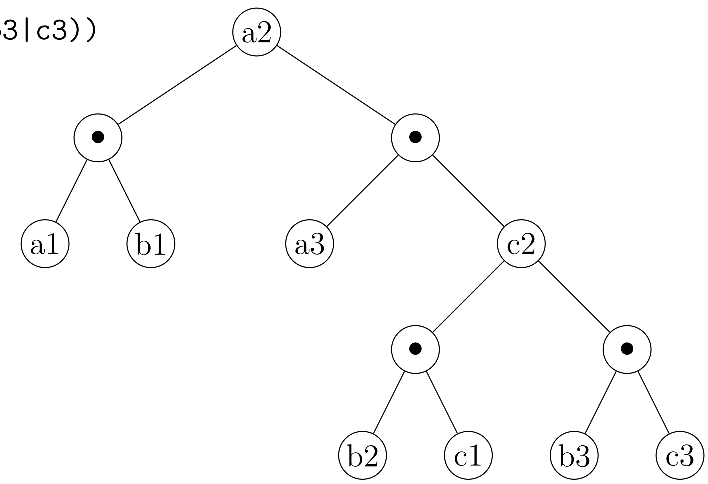


15. In each case, give this hex strategy in and-or form and draw the corresponding and-or tree:
a) from the diagram, the 1st-player Black win-strat
b) after 1.B[a2], the remaining 2nd-player Black win-strat
c) after 1.B[a2] 2.W[a3], the remaining 1st-player Black win-strat

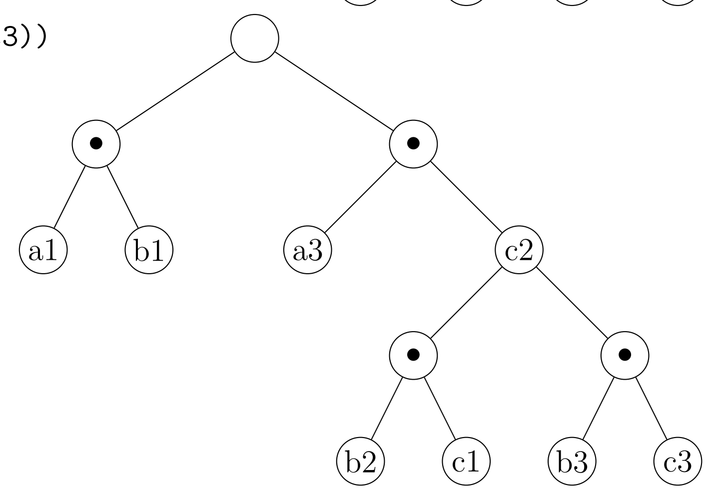


Answer.

a) `a2.(a1|b1).(a3| c2.(b2|c1).(b3|c3))`



b) `(a1|b1).(a3| c2.(b2|c1).(b3|c3))`



c) `c2.(a1|b1).(b2|c1).(b3|c3)`