**cmput 355 2024    homework 6, with hints**

Explain each answer, show all work. Unless stated otherwise,
Rose is the row player, Colin is the column player, matrix payoffs are for Rose.

1. We want to find the value of this matrix game.

```
-1  2 -1
 3  1  2
-2  1 -1
```

   a) Explain why Rose can ignore the last row as an action option.

   b) Using a), simplify the matrix.

   c) Using b), consider Colin, simplify the matrix.

   d) Using c), find a Von Neumann equilibrium of the reduced game.

   e) Using your answer to d), give a Von Neumann equilibrium of the original game.

2. Rose and Colin play this game.

```
 1  0 -2
 3  5 -4
-6  7  8
```

   a) What is Rose's expected payoff if she plays $S = (.7, .1, .2)$ and Colin plays $T = (.4, 0, .6)$? (Express your answer as an arithmetic expression: you do not need to simplify.)

   b) Either simplify the matrix as in the previous question or explain why this is not possible.

3. Rose and Colin play this game.

```
 1  2  1
 1  0  2
 3  1  0
```

   a) Give a linear program whose solution finds a Rose minimax strategy and Rose's minimax expected value.

   b) In what sense is this value minimax for Rose?

   c) Explain how to solve this LP using sagemath.

   d) Using sagemath, find a Von Neumann equilibrium for this problem.

4. Without using SageMath, find a Von Neumann equilibrium for this game.

```
0  1
5 -1
```

5. Rose and Colin play M, a version of rock-paper-scissors (RPS) with this matrix:

```
     rck ppr scr
rck   0  -1   2
ppr   1   0  -1
scr  -1   1   0
```

a) Explain why M favors Rose more than standard RPS.

b) Colin decides to play scissors with probability less that rock or paper. Why is this a good idea?

c) Colin decides to play scissors with probability 0. Find a Von Neumann equilibrium for this simplified game.

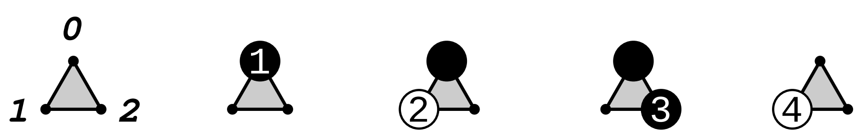
d) Find a Von Neumann equilibrium for M. Explain why Colin's decision in b) was ok but in c) was not best.

6. We could improve our github repo mcts hex player by using prior knowledge.

a) In mcts, explain what *using prior knowledge* means.

b) Give two examples of how to use prior knowledge in our github mcts hex player. For each example, give module name and line number and explain the changes you would make.

7. The game of go can be played on any graph, not just rectangular grids. For example, we can play go on a triangle, say with cells $\{0, 1, 2, \}$. *Trigo* is go played on a triangle. Here is a trigo game: 1.B[0] 2.W[1] 3.B[2] (captures the white stone) 4.W[1] (captures the two black stones) 5.B[pass] 6.W[pass] (game ends, W wins 3-0).




a) Give the number of legal trigo positions.

b) Draw all legal trigo positions. For each pair of positions $(x, y)$, draw an arc from $x$ to $y$ if there is some game in which there is a legal move from $x$ to $y$. We call this the *trigo transition diagram.*

c) Explain how to use the trigo transition diagram to count the number of legal trigo games.

**hints**

1. a) Each first row entry is at least as good as the corresponding second row entry

   b) remove the last row

   c) remove the first column

   d) discussed in detail in the lectures: Rose-strat (1/4, 3/4), Colin-strat (3/4, 1/4), value 5/4.

   e) Rose-strat (1/4, 3/4, 0), Colin-strat (0, 3/4, 1/4), value 5/4.

2. a)
```
(1st row) .7*.4* 1 + .7* 0* 0 + .7*.6* -2 +
(2nd row) .1*.4* 3 + .1* 0* 5 + .1*.6* -4 +
(3rd row) .2*.4*-6 + .2* 0* 7 + .2*.6*  8
```

   b) not possible: $M[1,1] < M[2,1]$ but $M[1,3] > M[2,3]$ so rows 1,2 are incomparable.

   you need to also show rows 1,3 incomparable, rows 2,3 incomparable, columns 1,2 incomparable, columns 1,3 incomparable, columns 2,3 incomparable.

3. Explained in the lecture.

   `https://webdocs.cs.ualberta.ca/~hayward/355/rps.pdf`

4. We want to find

$$\max_{0\le x,y\le 1,\, x+y=1}\{\ \min\ \{5y, x-y\}\}\ .$$

Substitute using $y = 1 - x$: if we don't find a solution, later we can try $y < 1 - x$. Now we want to find

$$\max_{0\le x\le 1}\{\ \min\ \{5-5x, 2x-1\}\}\ .$$

Plot the lines $f(x) = 5 - 5x$ and $g(x) = 2x - 1$ where $0 \le x \le 1$. From the plot, you can see that max $\{f(x), g(x)\}$ occurs where the lines cross, so

$$\begin{aligned} f(x) &= g(x) \\ 5-5x &= 2x-1 \\ 6 &= 7x \\ 6/7 &= x \end{aligned}$$

so $x = 6/7$, $y = 1 - x = 1/7$. So consider Rose stochastic strategy $S = (6/7, 1/7)$. Does this give us an equilibrium?

Against Colin's column 1 strategy, $S$ gives expected Rose payoff 5/7. Against Colin's column 2 strategy, $S$ gives expected Rose payoff 5/7. These payoffs are equal, so we are on the right track.

Next, left for you: find a stochastic Colin strategy $T = (a, b)$ with expected Rose payoff 5/7 for each of Rose's row-1 and row-2 strategies. (Repeat what we have done so far, except do it for Colin instead of Rose. Or use trial and error.)

An equilibrium is Rose $S = (6/7, 1/7)$, Colin $T = (2/7, 5/7)$, value 5/7.

5. a) for each possible R,C action pair, Rose does at least as well in the new game; for one action pair, she does better

   b) if Colin plays stochastic $(a, b, c)$, his worst case performance is max{ $2c - b$, $a - c$, $b - 1$ }: when we find an equilibrium, we will see that when this is minimized, $c$ is less than 1/3.

   c) here is the matrix (left) and then simplified (right).

```
z  rck  0 -1
x  ppr  1  0        1 0
y  scr -1  1       -1 1
```

   Rose wants

$$\max_{0 \le x, y \le 1,\, x+y=1} \{ \min \{x - y, y\}\} .$$

   substituting $y = 1 - x$, Rose wants

$$\max_{0 \le x, y \le 1} \{ \min \{2x - 1, 1 - x\}\} .$$

   so $x = 2/3$, $y = 1/3$, $a = 1/3$, $b = 2/3$, $z = 0$, $c = 0$, game value 1/3

   d) from Sagemath: Rose (1/4, 5/12, 1/3), Colin (1/3, 5/12, 1/4), value 1/12.

   Colin's minimax strategy has scissor probability less than that of rock and scissors – as suggested in b), but positive – not as suggested in c).

6. a) prior knowledge refers to adding knowledge to a leaf when it is first explored: instead of picking a random leaf to explore, pick the leaf whose prior knowledge suggests it is the best choice.

   b) mcts1.py lines 198, 199

```
for child in node.children:  # 198
    if child.sims == 0: # 199 unexplored children have priority
```

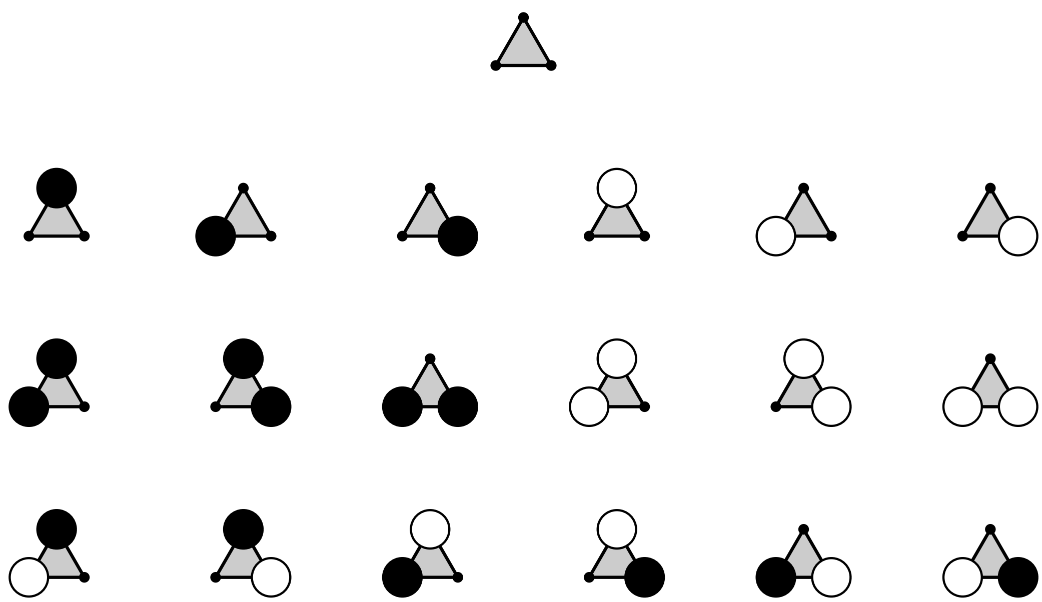
move 198, instead of processing children in default python order (will probably be by cell number, but could vary depending on python implementation), process children based on some prior knowledge estimation of move strength.

move 199: why favour unexplored children if they have a sibling with a really high win rate? e.g. if some sibling has won all simulations, and if the number of simulations is less than some threshold (perform an experiment to find a good threshold), then go with the winning sibling instead of an unexplored sibling.

another possible example: the simulations (see def rollout in mcts0.py) are all uniform random: surely we can do better. e.g. sometimes, use prior knowledge to pick the first move in a simulation.

7. a) a trigo position is legal if and only if it has at least one empty cell. there is 1 position with 3 empty cells, 3*4 positions with exactly 1 empty cell, 3*2 positions with exactly 2 empty cells, so total of 19 legal trigo positions.

   b) Here are the positions. We leave it to you to add the transition arcs.



Hint on next page.

Hint: from the empty position there are 0 in-arcs and 6 out-arcs, one to each of the positions in row 1. Label the rest of the positions by row and column number, e.g. (1,3) is in row 1 (count rows from top) and column 3 (count columns from left).

(1,1) has 2 in-arcs: from empty and (2,6).

(1,1) has 4 out-arcs: to (2,1), (2,2), (3,1), (3,2).

(2,1) has 2 in-arcs: from (1,1) and (1,2).

(2,1) has 1 out-arc: to (1,6).

(3,1) has 2 in-arcs: from (1,1) and (1,5).

(3,1) has 2 out-arcs: to (2,2) and (2,6).

You can find the rest of the arcs using symmetry.

c) A trigo game is legal if and only if there is a path (no repeated nodes) that starts at the empty position and follows arcs in the transition diagram: the order of moves in the game corresponds to the order of positions Thus to count the number of legal trigo games it suffices to count the number of such paths.