1. Find all winning moves from nim position (2 16 21 24). **Show your work.** You might not need all lines.

   Remove ______ stones from the pile with 2.

   Remove ______ stones from the pile with 16.

   Remove ______ stones from the pile with 21.

   Remove ______ stones from the pile with 24.

2. **(fill in the blanks) Claim.** For 16×16 Hex, P1 (the first player) has a winning strategy. **Proof.** Assume that P2 (the second player) has a non-losing strategy $S$. Then P1 can use $S$ like this: make a first move anywhere, then pretend that __________________________________________________________

   ________________________________________________________________

   and follow $S$ (so the next move she makes will be the first move in $S$). $S$ might sometimes require P1 to move to a cell where she already has a stone: in this case, she should ____________________________.

   So P1 follows $S$ and does not lose. Also, P2 follows $S$ and does not lose. This is a contradiction, because __________________________________________________________________________________.
3. **(fill in the blanks)** Claim. The stone at \(c2\) safely joins the bottom:

even if White plays next, Black can force the connection.

**Proof:** If White plays at any cell in \{ \__________ \}, Black replies at cell \__________

and so \_________________________________________

If White plays at any cell in \{ \__________ \}, Black replies at cell \__________

and so \_________________________________________

4. **(fill in the blanks)** This game starts 1.B[\(a3\)]. Black threatens to play 3.B[\(a2\)]

and then safely win with cells \{ \________ \}, or to play 3.B[\________] and safely

win with cells \{ b1, c1 \}. So White must play 2.W[\(b1\)], the only move that

interferes with both threats. But then Black plays 3.B[\________] which wins

because

\_________________________________________

\_________________________________________
hints

1. run `simple/nim/nimbigs.py` for the answer.

2. P1 can pretend that she is P2 by ignoring the first move she made. play anywhere (if Hex allowed the pass move, she could pass, but Hex does not allow the pass move). contradiction because Hex has no draws.

3. this is the classic 4.3.2 side connection: B has two semiconnections from c2 to the bottom: play b3 and then use one of {a4,b4}; play c3 and then use one of {d2,c3} and one of {c4,d4}. The cells sets of these semiconnections — {b3,a4,b4} and {c3,d2,c3,c4,d4} — do not intersect, so the two semiconnections form a virtual connection.

4. Black has a winning semiconnection: play c1 and use cells {b2,c2,b3,c3}