1. The function below is from a Hex program.
   (i) The function returns True if and only if ________________________.
   (ii) Explain the role of each variable: s ptm mvs j s[j] ECH optm k t.
   (iii) Why does the assertion hold?
   (Hint: the program also includes a function has_win.)
   (iv) Fill in blanks for (B), (C).
   (v) Make one change to the function that usually reduces the total number
calls to can_win( ) generated by can_win(s, ptm).

   ```python
def can_win(s, ptm):  # assume neither player has won yet
    mvs = []
    for j in range(N):
        if s[j]==ECH: mvs.append(j)
    assert(len(mvs)>0)
    optm = oppCH(ptm)
    for k in mvs:
        t = make_move(s, k, ptm)
        if has_win(t, ptm):
            return True
        if not can_win(t, optm):
            ____ (B) ____
            ____ (C) ____
```

2. We have seen that arbitrary tic-tac-toe and 3×3 Hex positions can be solved
   by simple minimax search algorithms that generate proof trees that usually
   have fewer than 10 000 nodes. Explain why this is possible for these two
games but not for 3×3 Go.

3. Watch the movie AlphaGo. Explain the role of these people or companies
   in the story: Aja Huang, David Silver, Fan Hui, Lee Sedol, Demis Hass-
   abis, Google, DeepMind. Explain how the AlphaGo algorithm works. With
   hindsight, how might LS have better prepared for the match. Describe the
two most exciting moves in the match (one in game 2, one in game 4), why
they were unexpected, and how they were created.

   there is another page
4. In rock-paper-scissors, Alice follows equiprobably: on each turn, play each move with probability 1/3.

If Bob plays rock against Alice, then Alice’s expected score is $1/3 \times 0 + 1/3 \times -1 + 1/3 \times 1 = 0$, since she plays each of rock/paper/scissors with probability 1/3.

(i) Show that Alice’s expected score is 0 for each of the other two possible moves by Bob.

(ii) Prove that there is no strategy that gives Alice a higher expected score, against all possible opponent strategies. (Hint. Bob can use Alice’s strategy.)

5. Modify rock-paper-scissors so that Alice’s payoff matrix is shown below: each row corresponds to a move by Alice, each column corresponds to a move by her opponent Bob. Assume that the game is zero-sum (so Bob’s payoff is the negative of Alice’s). For example, if Alice plays rock and Bob plays scissors, then Alice gets 2 points and Bob loses 2 points. (Another way to think of this is that Bob pays Alice $2.)

\[
\begin{array}{ccc}
R & P & S \\
R & 0 & -1 & 2 \\
P & 1 & 0 & -1 \\
S & -2 & 1 & 0
\end{array}
\]

(i) Explain why this game is fair, namely, why it is equally advantageous for both players.

(ii) Assume Alice again plays each possible move with probability 1/3. What is her expected win rate if Bob plays rock? If Bob plays paper? If Bob plays scissors? Against this strategy by Alice, assume Bob plays R/P/S with respective probabilities r/p/s. (So each of r,p,s is non-negative and their sum is one.) What is Bob’s best choice for r/p/s?

(iii) For this payoff matrix, assume Alice plays R/S/P with respective probabilities x/y/z. Give Alice’s best choice for x/y/z. Justify briefly.

6. Read this handout and then answer the following questions.


i) Prove that the first player minimax score for 1×4 Go is +4.

ii) Show a 1×4 Go game where the final first-player score is −4.

iii) For the first two Go positions in section 3x3 Go, explain why the final minimax score for Black will be +9.