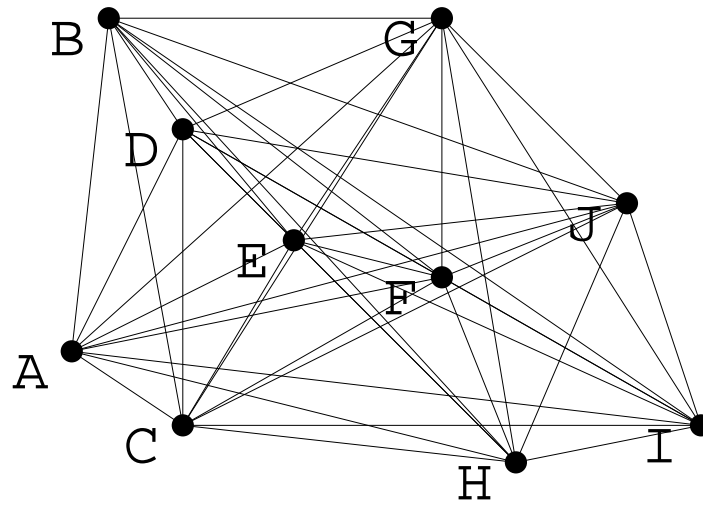


tsp 2-approx algorithm

- \* duplicate MST
- \* take 2-MST Eulerian tour
- \* shortcut tour (skip repeated nodes)  
=> Hamiltonian cycle MSTHC

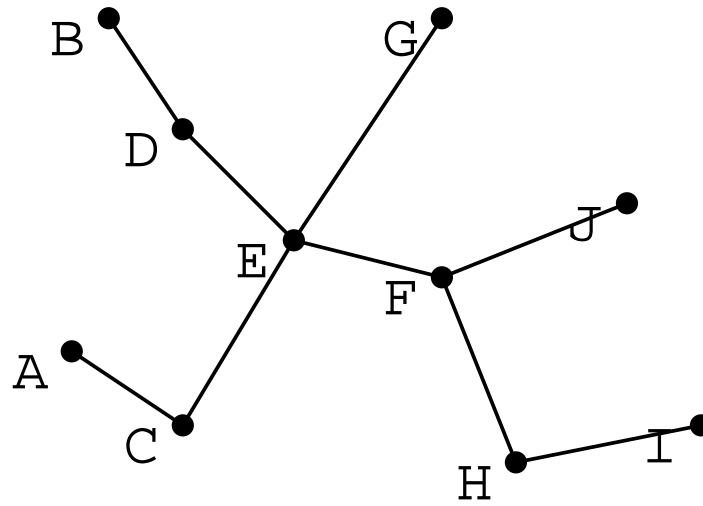
correctness

- \*  $\text{wt}(\text{MSTHC}) \leq 2 \cdot \text{wt}(\text{opt cycle})$



	A	B	C	D	E	F	G	H	I	J
A		5	2	4	4	6	7	7	9	9
B			6	2	4	6	5	9	11	8
C				4	3	4	7	5	8	7
D					2	4	4	7	9	7
E						2	4	5	7	5
F							4	3	4	3
G								7	7	4
H									3	4
I										4

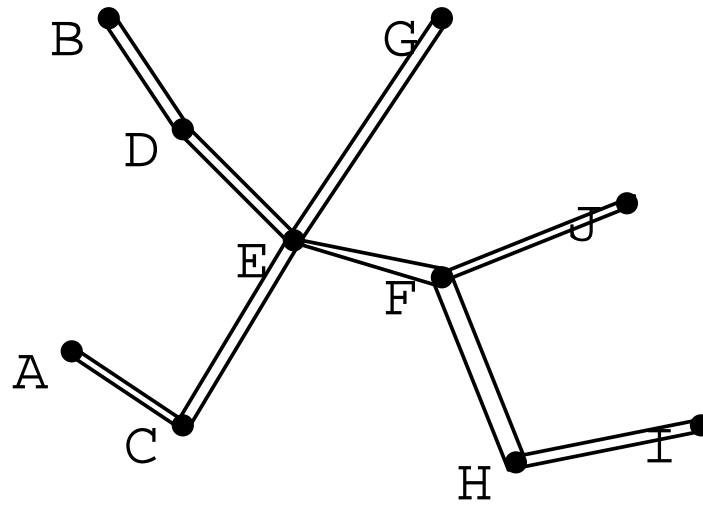
input: complete weighted graph



	A	B	C	D	E	F	G	H	I	J
A		5	2	4	4	6	7	7	9	9
B			6	2	4	6	5	9	11	8
C				4	3	4	7	5	8	7
D					2	4	4	7	9	7
E						2	4	5	7	5
F							4	3	4	3
G								7	7	4
H									3	4
I										4

find an MST

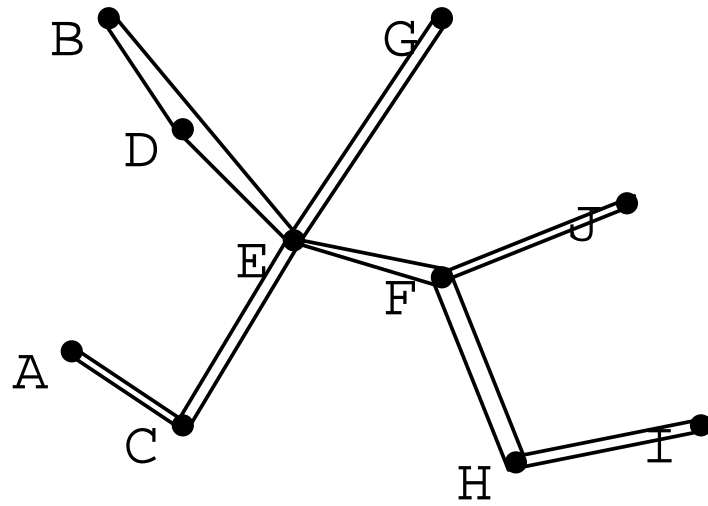
weight 24



initial Hamiltonian tour:

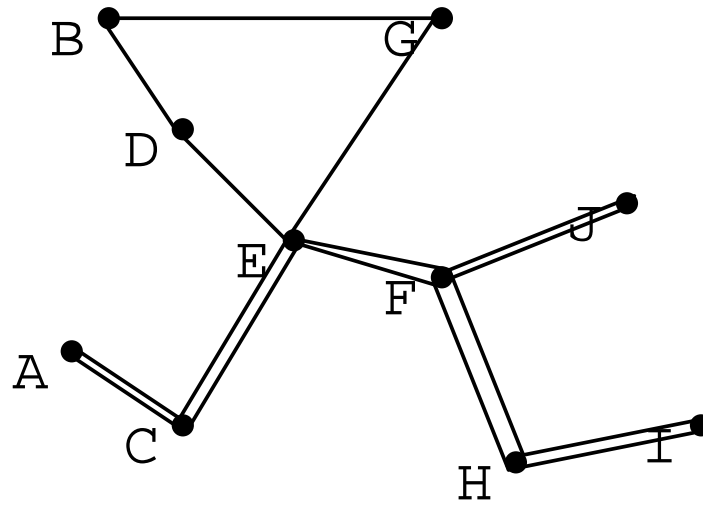
any duplicate-MST-Eulerian-tour

A C E D B D E G E F J F H I H F E C A



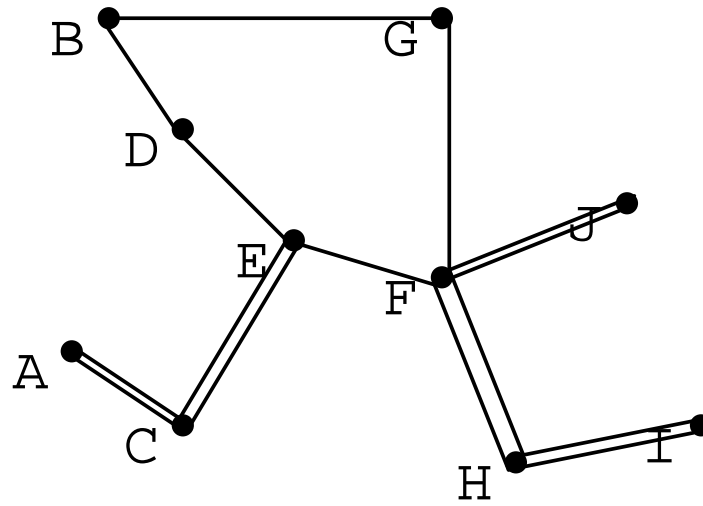
1st shortcut: skip repeated D ...

A C E D B    E G E F J F H I H F E C A



1st shortcut: ... and skip repeated E

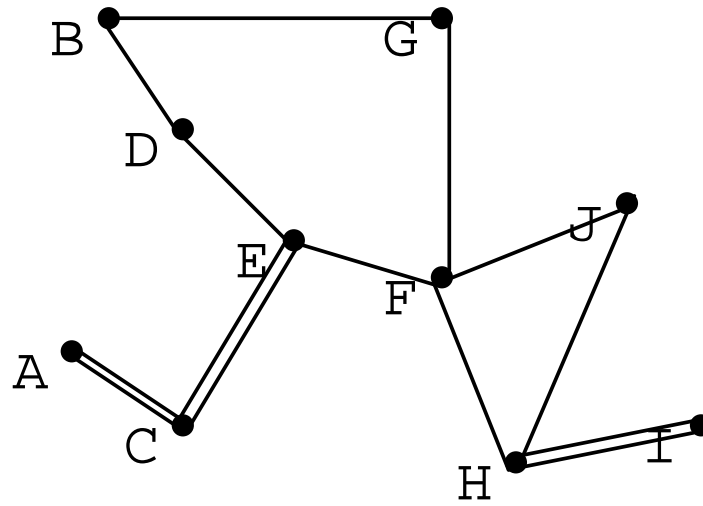
A C E D B      G E F J F H I H F E C A



2nd shortcut

A C E D B

G F J F H I H F E C A



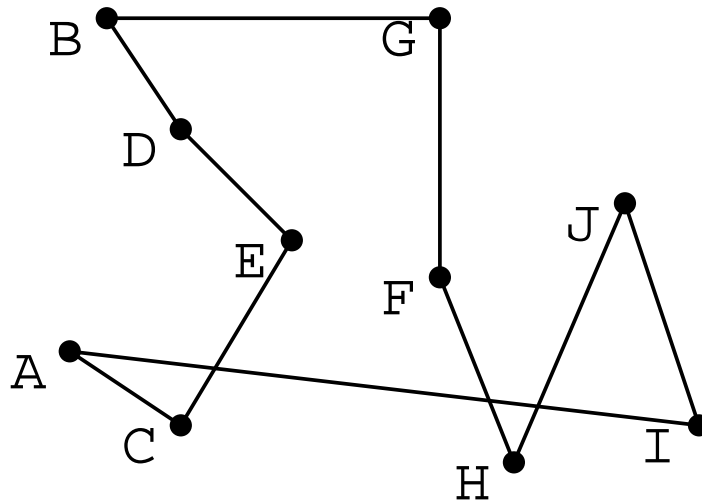
3rd shortcut

A C E D B

G F J

H I H F E C A





4th shortcut

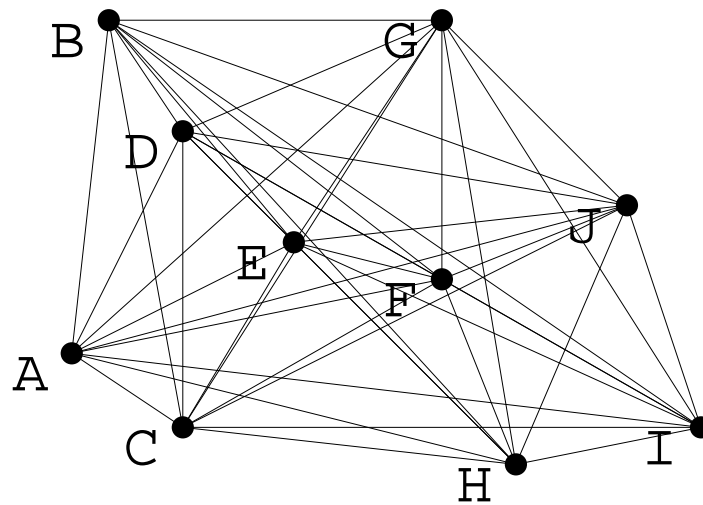
A C E D B      G    F J    H I                  A

tour now cycle (no repeated node): done

this Hamiltonian cycle is

at-most-2 approx



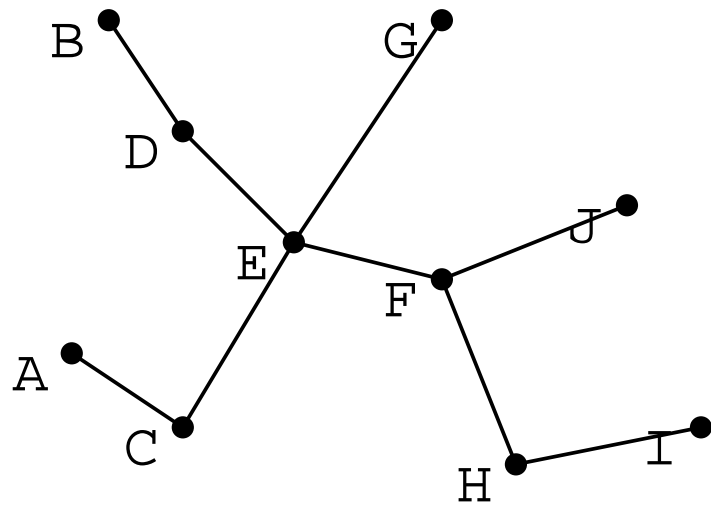


tsp 1.5-approx algorithm

- \* MST
- \* M: min matching on odd-degree-nodes of MST
- \* shortcut MST + M  
     => Hamiltonian cycle MSTMHC

correctness

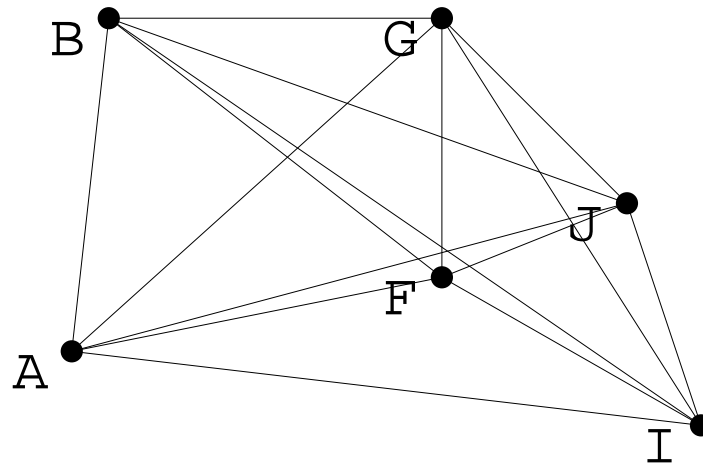
- \*  $\text{wt}(\text{MSTMHC}) \leq 1.5 \cdot \text{wt}(\text{opt cycle})$



	A	B	C	D	E	F	G	H	I	J
A		5	2	4	4	6	7	7	9	9
B			6	2	4	6	5	9	11	8
C				4	3	4	7	5	8	7
D					2	4	4	7	9	7
E						2	4	5	7	5
F							4	3	4	3
G								7	7	4
H									3	4
I										4

find an MST

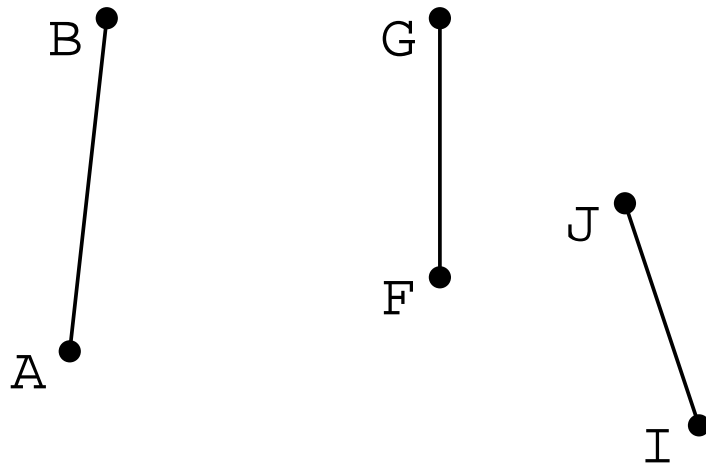
weight 24



complete subgraph induced by MST-odd-deg nodes:

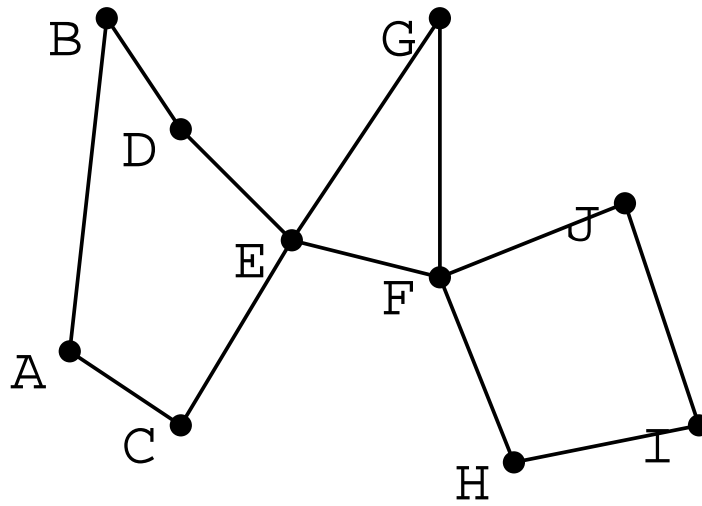
find min weight perfect matching M

	A	B	F	G	I	J
A		5	6	7	9	9
B			6	5	11	8
F				4	4	3
G					7	4
I						4



	A	B	F	G	I	J
A		5				
B					11	8
F				4	4	3
G					7	4
I						4

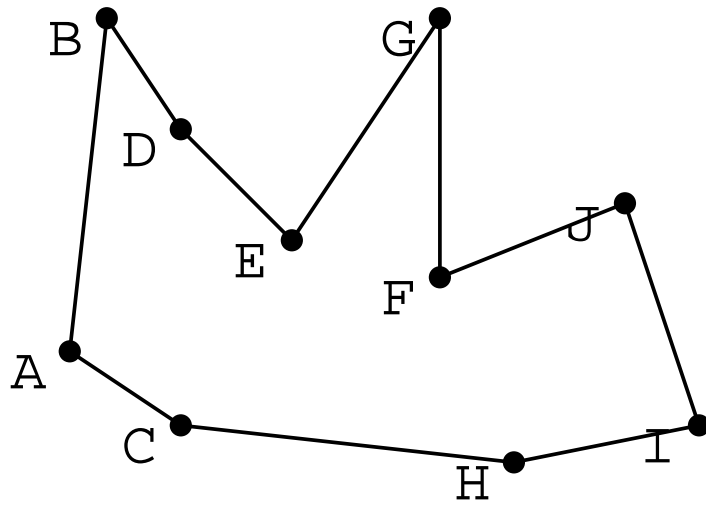
min-weight perfect matching M wt 13



	A	B	C	D	E	F	G	H	I	J
A		5	2	4	4	6	7	7	9	9
B			6	2	4	6	5	9	11	8
C				4	3	4	7	5	8	7
D					2	4	4	7	9	7
E						2	4	5	7	5
F							4	3	4	3
G								7	7	4
H									3	4
I										4

MST + M

wt 37

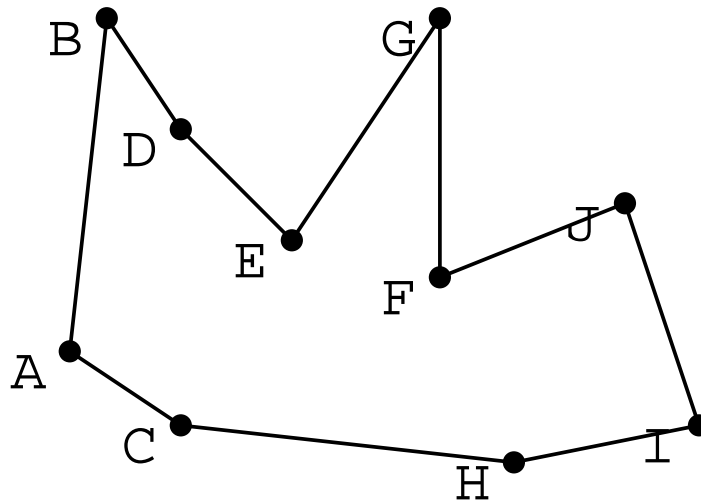


	A	B	C	D	E	F	G	H	I	J
A		5	2	4	4	6	7	7	9	9
B			6	2	4	6	5	9	11	8
C				4	3	4	7	5	8	7
D					2	4	4	7	9	7
E						2	4	5	7	5
F							4	3	4	3
G								7	7	4
H									3	4
I										4

shortcut MST+M

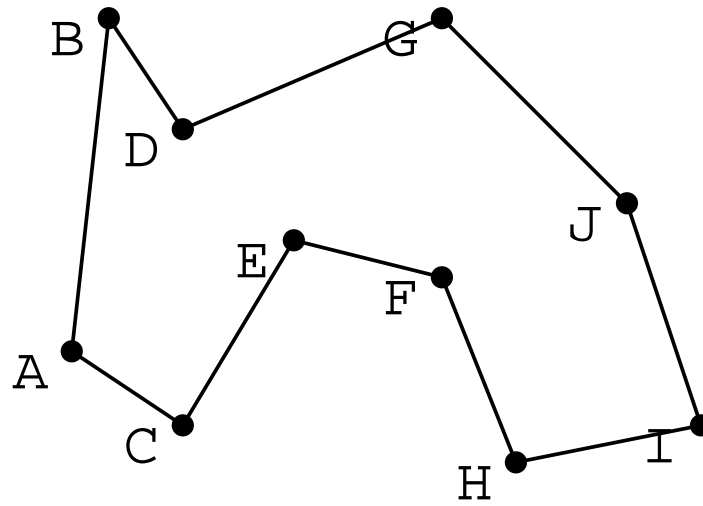
34





correctness

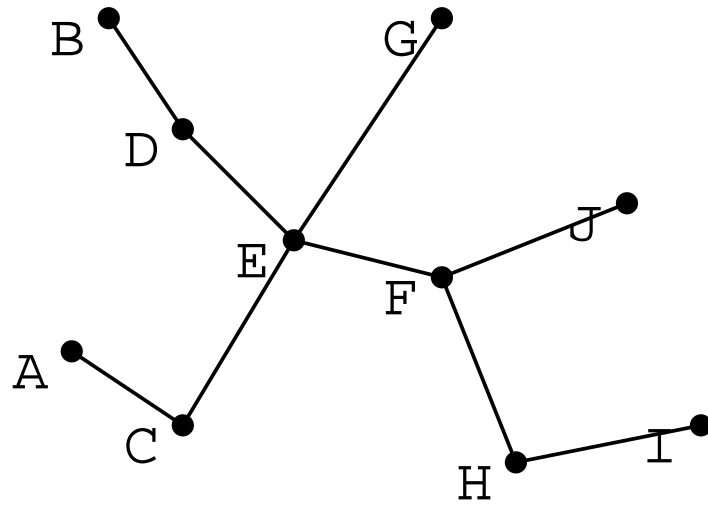
- \*  $\text{wt}(\text{MST}) \leq \text{wt}(\text{graph Hamiltonian cycle})$       why?  
   : opt-cycle minus edge is spanning tree
- \*  $\text{wt}(\text{MST-odd-deg Ham-cycle}) \leq \text{wt}(\text{graph Ham-cycle})$   
   : delete nodes, shortcut
- \*  $\text{wt}(\text{MSTod min-wt perf-match}) \leq \text{wt}(\text{MSTod Ham-cycle})/2$   
   : MSTod Ham-cycle partitions into M1 and M2  
   : min of these has  $\text{wt } m_0 \leq \text{wt}(\text{MSTod Ham-cycle})/2$   
   :  $\text{wt}(\text{MSTod min-wt perf-match}) \leq m_0$



	A	B	C	D	E	F	G	H	I	J
A		5	2	4	4	6	7	7	9	9
B			6	2	4	6	5	9	11	8
C				4	3	4	7	5	8	7
D					2	4	4	7	9	7
E						2	4	5	7	5
F							4	3	4	3
G								7	7	4
H									3	4
I										4

optimal cycle (brute force)

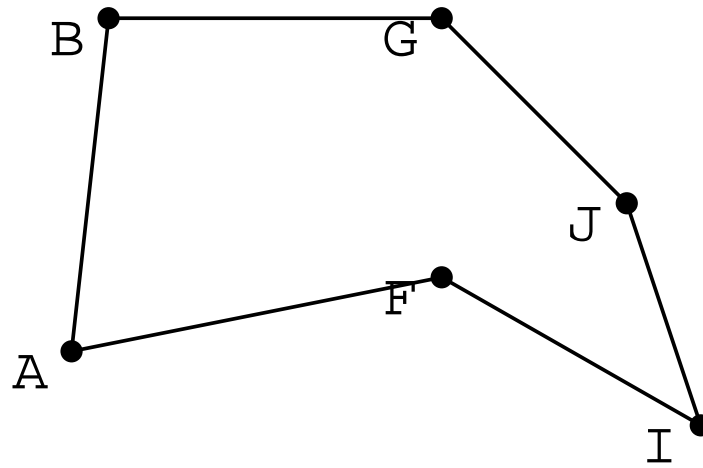
32



an MST

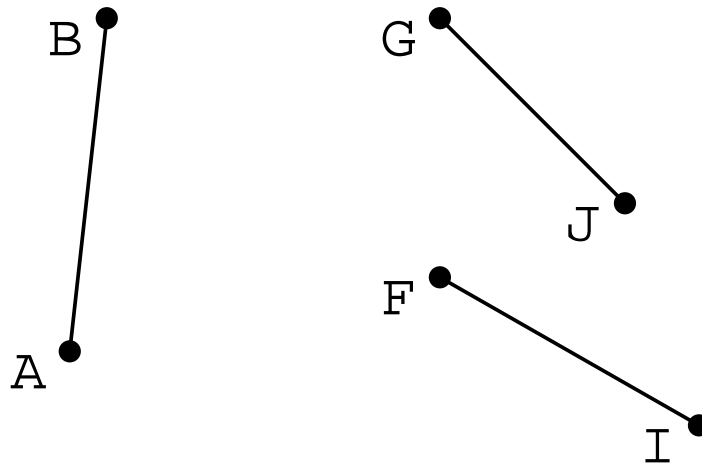
weight 24

odd-degree nodes A B E G J I



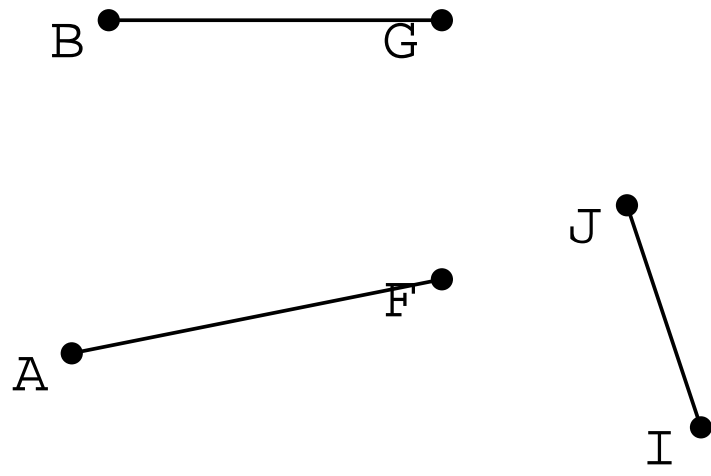
	A	B	F	G	I	J
A		5	6	7	9	9
B			6	5	11	8
F				4	4	3
G					7	4
I						4

MST-odd-deg-nodes:      opt-cycle      wt 28



	A	B	F	G	I	J
A		5				
B						
F				4	4	3
G					7	4
I			6	7		9
J			6	5	11	

MST-odd-deg-nodes: opt-cycle, matching 1, wt 13



	A	B	F	G	I	J
A		5	6	7	9	9
B			6	5	11	8
F				4	4	3
G					7	4
I						4

MST-odd-deg-nodes: opt-cycle, matching 2, wt 15