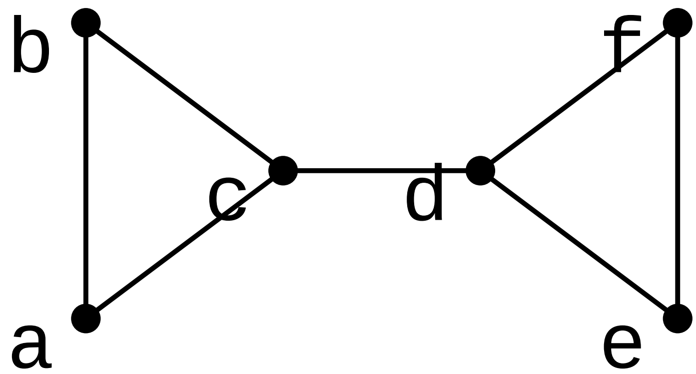
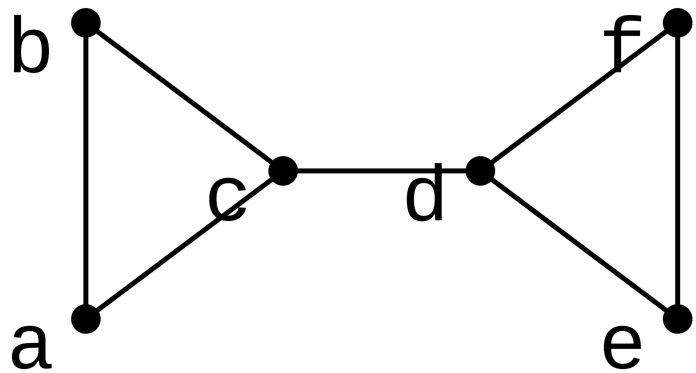
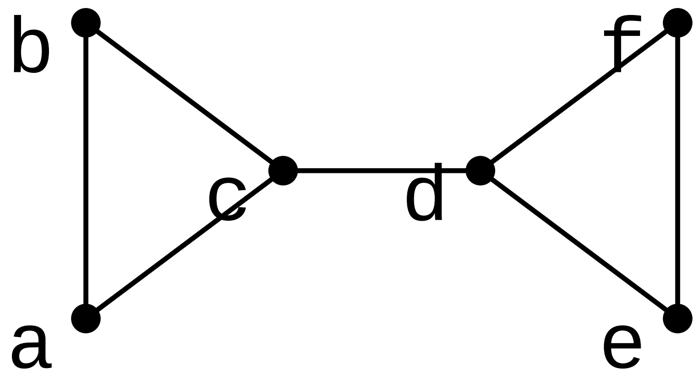


min cut

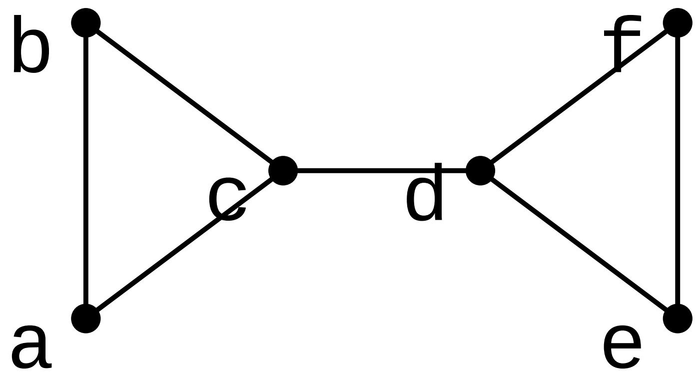
```
* unweighted graph G, n nodes

* cut: bipartition of node set
     e.g. {a,d} {b,c,e,f}

* an edge *crosses* a cut A,B
     if it has one end in each part
     e.g. {a,b}, {a,c}, {c,d}, {d,f}, {d,e}

* cut size: number of edges that cross it
     e.g. 5

* min cut problem: given G, find any min cut
```



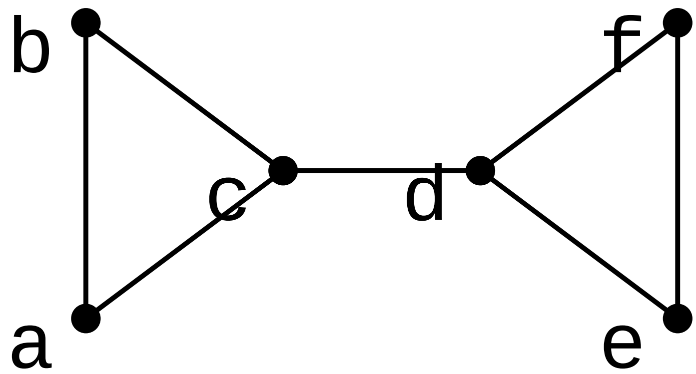
brute force min cut alg'm ?




```
randomized kruskal min cut

  * unweighted graph, n nodes
  * consider edges in uniform-random order
  * apply Kruskal MST until exactly 2 components
  * prob(this cut X is a min cut) >= 2/(n(n-1))  (1)

  proof of (1) ?
```



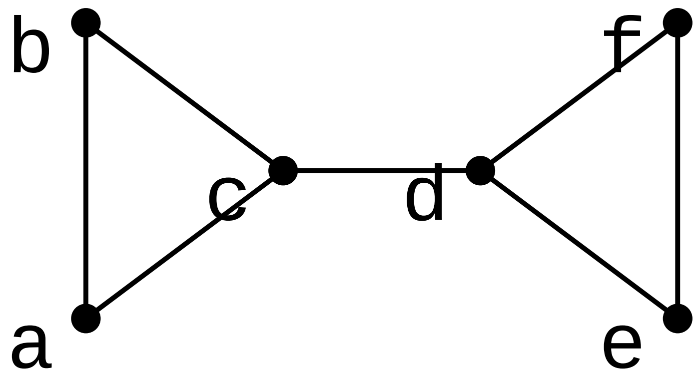

```
above example

  * prob(RKMC cut is min) >= 2/(6*5) = 1/15 = .0666...

  * prob(                 )  =   ?
     * unique min cut   {{a, b, c}, {d, e, f}}  Y
     * consider all 7! edge permutations
     * RKMC(permutation) returns Y iff
        - in perm'n, edge cd is last, or
        - in perm'n, edge cd is 2nd-last, or
        - in perm'n, edge cd is 3rd-last and last
          two edges are from different triangles
     * prob = 1/7 + 1/7 + (1/7)(3/5)     exercise

             = 13/35 = .3714...
```



* call RKMC t times, take smallest cut found
* prob(best cut found is min cut) ?

t	1.0 - (1.0 - 13/35)^t	
1	.371	
2	.604	with bound (1)
3	.751	67 trials
4	.843	prob >= .99
5	.901	
6	.938	with n = 100
7	.961	bound (1)
8	.975	22794 trials
9	.984	prob >= .99
10	.990	

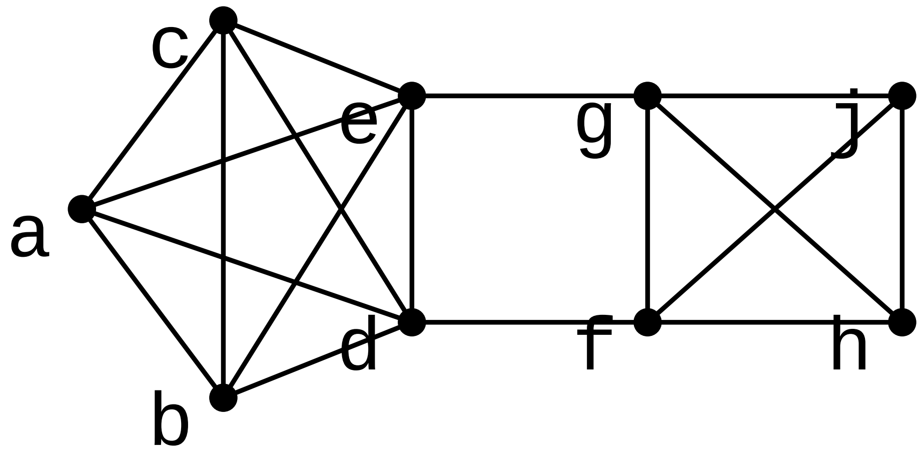
after 10 trials
prob(min cut) >= .99



proof of (1)

- * F is RKMC forest-so-far
- * after t steps, F has k = n-t components
- * number edges leaving component Cj of F ?
- * consider min cut Y, |Y| = y
- * { Cj, V - Cj} is a cut, so
  at least y edges leave Cj
- * so number edges between components is ky/2
- * edges between components are exactly the
  RKMC-eligible edges
- * prob(next edge picked is in Y)
  <= y / (ky/2) =   2/k
- * prob(RKMC picks no edge from Y)

```
      n-2 n-3 n-4     3 2 1     2
   >= --- --- --- ... - - - = ------
       n  n-1 n-2     5 4 3   n(n-1)
```



exercise