Assignment 3 CMPUT 272 Fall 2013 Posted: ??
Due before 10:00am, October 30
in box labeled "CMPUT 272 Fall 2013" (opposite room CSC 145)

(Mandatory assignment cover-sheet; without it, your work will not be marked.)

Submitting student: ______________________ Section ID (circle): **B1, EB1, B2, EB2**

Collaborating classmates: ____

Other collaborators: ____

References other than the textbook and handouts:

____

____

____

____

Regardless of the collaboration method allowed, you must always properly acknowledge the sources you used and people you worked with. Your professor reserves the right to give you an exam (oral, written, or both) to determine the degree that you participated in the making of the deliverable, and how well you understand what was submitted. For example, you may be asked to explain any solution that was submitted and why you choose to write it that way. This may impact the mark that you receive for the deliverable.

So, whenever you submit a deliverable, especially if you collaborate, you should be prepared for an individual inspection/walkthrough in which you explain what every line of assignment does and why you choose to write it that way.

1. Prove or disprove:

   (a) If $n \in \mathbb{N}^+$ is composite, and there is no integer t such that $n = t * t$, then there exists a prime p such that $p \mid n$ and $p^3 \leq n$.

   (b) For any predicate P with domain D^2, $\neg(\exists b \in D, \forall a \in D, P(a,b)) \Rightarrow \neg(\forall x \in D, \exists y \in D, P(x,y))$.

2. Prove or disprove:

   (a) $\log_3 2$ is a rational number.

   (b) For all $n \in \mathbb{N}^+$, $\sum_{j=1}^{n} 1/(2^j) = 1 - 2^{-n}$.

3. Here is a 2-player alternate-turn board game: the board is a row of n squares; each player has dominoes; on a turn, a player places a domino so that it covers 2 adjacent squares; the loser is the first player who cannot place a domino.

   Example. Let $n = 6$. Label squares $\{1, 2, 3, 4, 5, 6\}$, so that consecutively labelled squares are adjacent. Bob covers squares 2,3. Sue covers squares 4,5. Bob cannot play, so he loses.

   (a) For each n in $\{1, 2, \ldots, 6\}$, determine whether the first player has a winning strategy.

   (b) Prove that the first player has a winning strategy if n is even.

4. Let $t(n) = 3^n - 2$. Here is the definition of a *tersenne number*: $z \in \mathcal{Z}, z \geq 2, \exists n \in \mathcal{Z}, z = t(n)$.

   (a) Using only words, give the definition of a tersenne number.

   (b) A *tersenne prime* is a tersenne number that is prime. Prove or disprove: every tersenne number is a tersenne prime.

   (c) Using only symbols (including $\mathcal{P}$ for the set of primes), give a statement equivalent to this: there are an infinite number of tersenne primes.

   (d) Prove or disprove: for all integers $n \geq 3$, $n \equiv 3 (\text{mod } 4) \Rightarrow t(n) \equiv 0 (\text{mod } 5)$.

5.
```
def mydiv(n,d):      # assume n>=0, d>0
  q,r = 0,n          # q assigned 0,  r assigned n
  while              # at this point, variant: r
                     # at this point, invariant: n == q*d + r
      r >= d:        # r >= d ?
    q,r = q+1, r-d   # if yes, then q ass. q+1, r ass. r-d
  return q,r
```

   For the function `mydiv(n,d)`, prove:

   (a) the variant holds,

   (b) the invariant holds,

   (c) for every non-negative integer n and positive integer d, the returned values q, r satisfy $n = q*d+r$,

   (d) the value r returned by the program satisfies $0 \leq r < d$.

   Assume that someone writes another program that, for $n = 987371$ and $d = 9871$, returns integers x and y such that $n = x * d + y$, where $0 \leq y < d$.

   (e) Prove that the values q, r returned by `mydiv(n,d)` equal x, y respectively.

6.
```
def ctz(n):
  while n>1:
    print n,
    if 0==n%2:          # 0 equals n (mod 2) ?
      n = n/2           # n assigned value floor(n/2)
    else:
      n = 3*n+1         # n assigned value 3*n+1
  print "1\n"

for j in range(2,11):  # j ranges from 2 to 10
  ctz(j)                #   execute ctz(j)
```

Hint: execute — and alter as necessary — this python program.

(a) Prove or disprove. For all positive integers t, `ctz(t)` terminates after at most t loop iterations.

(b) Prove or disprove. Let x be the smallest positive integer for which `ctz(x)` never terminates. Then $x \equiv 3 (\text{mod } 4)$.

(c) For which integer $x < 30$ does `ctz(x)` make the most loop iterations? For this x, how many does it make?

7. By induction on e, for $1 \leq e$, for any integer a, prove that this function returns a^e.

```
def exp(a,e):          # assume 1 <= e
  if e == 1:
     return a
  elif 0 == e%2:                # 0 equals e (mod 2) ?
     return exp(a, e/2)**2      # return exp(a, floor(e/2)) squared
  else:
     return a*exp(a, e/2)**2    # return a * "                    "
```

8. (i) Complete as much of the following sudoku puzzle as you can **without making any guesses**. For the first 5 cells filled, explain your deduction. E.g. row 7, column 7 is 9 by crosshatching. Hint: you can complete all but 15 cells; for each of these, the only possible values are in {4,5,8}.

	1	3				6	9	
				2			3	7
6								
	4		7				5	
9			3		5			8
	2				4		6	
								6
1	3			9				
	9	2		7	1			

(ii) Finish solving the puzzle. Explain how you deduced the next cell entry.

	1	3				6	9	
				2			3	7
6								
	4		7				5	
9			3		5			8
	2				4		6	
								6
1	3			9				
	9	2		7	1			

(iii) Prove or disprove: this puzzle has exactly one solution.