CMPUT 272 Fall 2007 Midterm 1 Version B

50 minutes 15 marks no computing devices answer in the space provided show your work

1. [2 marks]

   (i) Express 4016053_9 as a sum of powers of 9.

   (ii) Express 1011100111_2 in hexadecimal.

   (iii) Express 132_{10} in base 3 (show your work).

2. [3 marks] Consider an 8-bit two's complement register.

   (i) Give the decimal number x represented by 1 1 1 0 1 1 0 1 .

   (ii) Show how the decimal number 28 would be represented.

   (iii) Show the register contents after the operation "$x + 28$", where x is from part (i).

   (iv) Does this operation cause overflow? Explain briefly.

3. [2.5 marks] Let $T(n)$ be the predicate "if n is even then n+5 is odd".
   (a) State the contrapositive of $T(n)$.

   (b) State the converse of $T(n)$.

   (c) State the negation of $T(n)$.

   (d) Prove: $\forall n \in Z, T(n)$.

4. [2.5 marks] Let $T(n)$ be the predicate $\sum_{j=2}^{n} 2^j = 2^{n+1} - 4$.

   Prove by induction that $T(n)$ holds for all integers $n \geq 2$.

   Base case:

   Inductive case:

5. [3.5 marks]

   (i) Using only symbols from $\{p, q, (,),\ , \Rightarrow, \Leftrightarrow\}$, express "$q$ is not sufficient for p".

   (ii) Using only symbols from $\{p, q, r, (,),\ , \vee, \wedge\}$, express "$r$, unless p or not q.

   (iii) Negate and simplify $\forall x, \forall y, \exists z, P(x, y, z) \Leftrightarrow T$

   (iv) Prove or disprove: $(p \Rightarrow q) \Rightarrow r$ is logically equivalent to $p \Rightarrow (q \Rightarrow r)$.

6. [1.5 marks]

   A sudoku solver has deduced the following top row entry possibilities:

2,4	3,7,8	1,5,6,9	2,4	1,5,6	3,4,8	3,7,8	3,7,9	1,5,6,9

   Later, the solver deduces that the row two, column four entry is 2. Show the resulting top row entry possibilities; reduce as much as possible. Justify briefly.
