final exam cmput 272 fall 2012 Prof Hayward

6 pages 3 hours 6 marks/page no electronics **show all work**

Give the base 3 representation of 48207_9.

Give the base 5 representation of 482.

Give the 8-bit twos-complement representations of the values shown.

89								
117								
-117								
$89-117$								

Let a be the statement form $p \wedge \neg q$.
Let b be the statement form $a \vee \neg r$.
Let c be the statement form $b \Rightarrow p$.
Complete the truth table.

p	q	r	a	b	c
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

A *literal* is either a variable or the negation of a variable. An *and-clause* is the conjunction of one or more literals. An *or-clause* is the disjunction of one or more literals. A boolean expression is in *disjunctive normal form* if it is the disjunction of one or more and-clauses. A boolean expression is in *conjunctive normal form* if it is the conjunction of one or more or-clauses.
The statement form $f(p, q, r)$ is given by the truth table.

p	q	r	f
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

Express $\neg f$ in disjunctive normal form.

Express f in conjunctive normal form.

```
#   w is remaining number of white beans
#   b is remaining number of black beans
#   removeBean() updates w,b correctly
#   putBlackBean() updates b correctly
while w + b > 1:
  bn1 = removeBean()
  bn2 = removeBean()
  if bn1.col != bn2.col:
    putBlackBean()
    putBlackBean()
```

bn1	bn2	change in w	change in b
black	black	0	-2
black	white		
white	black		
white	white		

Complete the above table, which shows the change in w and b after a loop iteration.

Find a variant for this loop. Justify briefly.

Assume to start that $w > 0$ and $w + b = 314159$.
Give all possible ordered pairs (w, b) of the final number of white and black beans. Justify briefly.

A binary relation on a set S is *inclusive* if every element in S relates to at least one element. For example, $\{(1,2),(2,3)\}$ is not inclusive, because 3 does not relate to any element. $I(n)$ is the number of inclusive relations on an n-set. Find $I(n)$ for $n = 1, 2, 3$ and for arbitrary n.

n	1	2	3	arbitrary
$I(n)$				

For these preferences between students {1,2,3,4,5} and universities {a,b,c,d,e}, find the stable matching that is best for universities. Show this matching by drawing lines in the box.

```
1: a b c d e     a: 1 2 3 4 5
2: c e d b a     b: 3 1 5 2 4
3: d c b a e     c: 4 3 2 1 5
4: b a c d e     d: 2 1 5 4 3
5: e a d b c     e: 3 1 2 5 4
```

1	2	3	4	5
a	b	c	d	e

Let $n \geq 3$ be the number of students and the number of universities, let no two students have the same preference sequence, and let no two universities have the same preference sequence. Prove/disprove: the stable matching that is best for students is different from the stable matching that is best for universities.

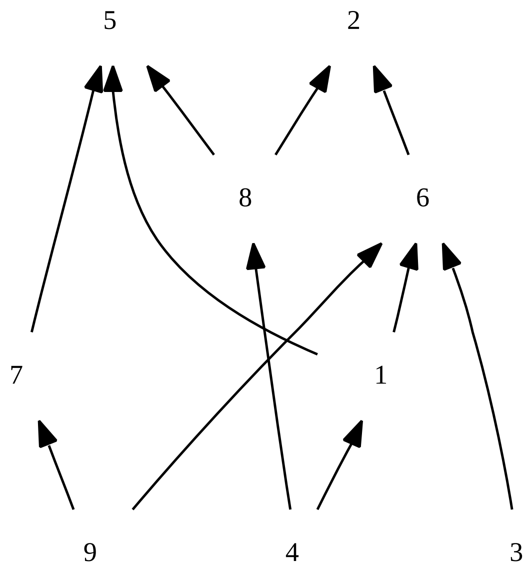
Recall that $S(n,k)$ is the number of k-partitions of an n-set.
Recall that $S(n,1) = S(n,n) = 1$ for $n \geq 1$.
Recall that $S(n,k) = S(n-1,k-1) + kS(n-1,k)$ for $n > k > 1$.
Use this information to prove by induction: for all integers $n \geq 2$, $S(n,n-1) = n(n-1)/2$.

The proof in the base case:

The inductive hypothesis in the inductive case:

The rest of the proof:

For the poset with this Hasse diagram, give a 4-chain and a partition of the poset into 4 antichains.



Give a 4-antichain and a partition of the poset into 4 chains.

Prove/disprove: there is a partition of the poset into 3 chains.