SHOW ALL YOUR WORK cmput 272 fall 2011 midterm (version B)
70 minutes 18 marks, 3 pages no computing devices answer in the space provided

1. Let $S(n) = \sum_{j=0}^{n} 5^j$. Let $f(n) = (5^{n+1} - 1)/4$. Prove by induction: $\forall n \in \mathcal{N}, S(n) = f(n)$.

   Base case:

   Inductive hypothesis:

   Rest of proof:

2. Using boolean algebra, prove $\sim (a \vee (b \wedge c)) \Leftrightarrow (\sim a \wedge \sim b) \vee (\sim a \wedge \sim c)$. Justify each step.

3. Is $\forall a \in \mathcal{Z}, \forall b \in \mathcal{Z}, S(a, b)$ logically equivalent to $\forall b \in \mathcal{Z}, \forall a \in \mathcal{Z}, S(a, b)$? Explain briefly.

4. Express the following boolean function $f(p, q, r)$ in DNF.

p	q	r	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

5. Express 749053_8 as a sum of multiples of powers of 8.

   Express 111111101011001_2 in hexadecimal.

   Express 761_{10} in base 5.

   For an 8-bit two's complement register:

   - give the maximum and minimum integers that can be represented

   - show how 23 and -46 are represented

   - show the register after the operation "23 + -46".

6. This loop executes on a tin that starts with $w^* \geq 0$ white beans and $b^* \geq 0$ black beans:

```
while tin has >= 2 beans
  bn1 := removeBean()    bn2 := removeBean()
  if ((bn1.col = BLACK) and (bn2.col = BLACK)) then putBlack()
  elsif (bn1.col <> bn2.col)                   then putWhite()
```

   Does the loop always terminate? Justify briefly.

   Assume that the loop terminates, and that B black beans and W white beans remain. What can you say about B and W in terms of b^* and w^*? Justify briefly.

7. Call an integer x *shiny* if there exists an integer y such that $x = 4y$.

   Let $S(a, b)$ be the predicate "if $a \times b$ is not shiny then a and b are not shiny".

   Let $T(a, b)$ be the predicate $\forall a \in \mathcal{Z}, \forall b \in \mathcal{Z}, S(a, b)$.

   State the converse of $S(a, b)$.

   State the contrapositive of $S(a, b)$.

   In words, simplified, state the negation of $T(a, b)$.

   Prove or disprove $T(a, b)$.