

**SHOW ALL YOUR WORK**    **comput 272 fall 2011**    **midterm (version A)**

70 minutes    18 marks, 3 pages    no computing devices    answer in the space provided

1. Let  $S(n) = \sum_{j=0}^n 3^j$ . Let  $f(n) = (3^{n+1} - 1)/2$ . Prove by induction:  $\forall n \in \mathcal{N}, S(n) = f(n)$ .

Base case:

Inductive hypothesis:

Rest of proof:

2. Using boolean algebra, prove  $\sim (a \wedge (b \vee c)) \Leftrightarrow (\sim a \vee \sim b) \wedge (\sim a \vee \sim c)$ . Justify each step.

3. Is  $\forall x \in \mathcal{Z}, \forall y \in \mathcal{Z}, S(x, y)$  logically equivalent to  $\forall y \in \mathcal{Z}, \forall x \in \mathcal{Z}, S(x, y)$ ? Explain briefly.

4. Express the following boolean function  $f(p, q, r)$  in DNF.

$p$	$q$	$r$	$f$
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

5. Express  $304956_8$  as a sum of multiples of powers of 8.

Express  $101110101111011_2$  in hexadecimal.

Express  $716_{10}$  in base 5.

For an 8-bit two's complement register:

- give the maximum and minimum integers that can be represented
- show how 25 and -50 are represented
- show the register after the operation "25 + -50".

6. This loop executes on a tin that starts with  $W \geq 0$  white beans and  $B \geq 0$  black beans:

```
while tin has >= 2 beans
  bn1 := removeBean()    bn2 := removeBean()
  if ((bn1.col = BLACK) and (bn2.col = BLACK)) then putBlack()
  elseif (bn1.col <> bn2.col) then putWhite()
```

Does the loop always terminate? Justify briefly.

Assume that the loop terminates, and that  $b^*$  black beans and  $w^*$  white beans remain. What can you say about  $b^*$  and  $w^*$  in terms of  $B$  and  $W$ ? Justify briefly.

7. Call an integer  $x$  *smooth* if there exists an integer  $y$  such that  $x = 3y$ .

Let  $S(a, b)$  be the predicate “if  $a \times b$  is not smooth then  $a$  and  $b$  are not smooth”.

Let  $T(a, b)$  be the predicate  $\forall a \in \mathcal{Z}, \forall b \in \mathcal{Z}, S(a, b)$ .

State the converse of  $S(a, b)$ .

State the contrapositive of  $S(a, b)$ .

In words, simplified, state the negation of  $T(a, b)$ .

Prove or disprove  $T(a, b)$ .