cmput 272 fall 2011 final exam

8 pages 3 hours 37 marks

no electronic devices write answers in the space provided show all work

1. [4 marks]

   represent 219 in binary

   represent 8701_9 in base 3

   represent 1011101010_2 in hexadecimal

   represent 101101_2 in decimal

   represent -219 in 10-bit two's complement

   represent 10-bit two's complement 1111011010 in decimal

2. [4 marks] complete the truth table for the boolean function $f = (p \rightarrow \sim r) \rightarrow ((q \vee\ r) \rightarrow p)$

p	q	r	f
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

represent $x \oplus y$ using symbols from $\{\wedge, \sim, (,), x, y\}$

3. [4 marks] let $g = \gcd(29{,}17)$

   use Euclid's algorithm to find g

   find integers x, y such that $29x + 17y = g$

   find the smallest nonnegative integer t such that $29t = g \bmod 17$

4. [5 marks] Recall: $\mathcal{Z}^+$ is the set of positive integers, and $a|b$ (a divides b) if there is an integer k such that $a \cdot k = b$.

   Let x, y be positive integers, let $D = \{d \in \mathcal{Z}^+, d|x \wedge d|y\}$, let $M = \{m \in \mathcal{Z}^+,\ \exists s, t \in \mathcal{Z},\ m = s \cdot x + t \cdot y\}$. Prove or disprove: $\forall d \in D, \forall m \in M, \exists t \in \mathcal{Z}^+, m = d \cdot t$.

   Let $x = 1321181, y = 1291929$. Notice: $x = 103 \cdot 12543, y = 103 \cdot 12827, 103 = 6278 \cdot y - 6139 * x$. Prove or disprove: 103 is the smallest positive linear combination of x, y.

5. [4 marks] Let p be the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 10 & 12 & 8 & 9 & 3 & 6 & 1 & 11 & 7 & 4 & 5 & 2 \end{pmatrix}$.

give p in cyclic form

give p^{99} in cyclic form

give the smallest integer t such that $p^t = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \end{pmatrix}$

6. [5 marks]

```
while tin has >= 2 beans
  bn1 := removeBean()    bn2 := removeBean()
  if (bn1 and bn2 are both white) then
     putBlack() putBlack() putBlack()
  elsif (bn1 and bn2 are both black)
     putBlack()
  else
     putWhite()
  endif
  #### comment: bn1 and bn2 are thrown away
endwhile
```

Let w_t, b_t be the number of white,black beans in the tin after the loop body executes exactly t times. So, the code starts with $w_0 \geq 0$ white beans and $b_0 \geq 0$ black beans. Justify each answer briefly.

Does the code always terminate? If yes, give a variant that holds on entering the loop body. If no, explain.

Assume that $w_0 = 21$, $b_0 = 15$, and the code terminates after the loop body executes exactly k times. Give w_k, b_k, and possible values for k.

7. [4 marks] $S(n,k)$ is the number of k-partitions of an n-set. List all 2-partitions of the 4-set $\{a,b,c,d\}$.

For all $n \geq 2$, $S(n,2) = S(n-1,1) + 2S(n-1,2)$; explain briefly why this is true. (In your answer, do **not** use the general recursive formula for S(n,k)).

8. [7 marks] prove by induction: for any real number a, for all integers $n \geq 0$,

$$\sum_{j=0}^{n} (a+j) \;=\; \frac{(2a+n)(n+1)}{2}$$

base case

inductive case

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