

1. Write a statement acknowledging **all** resources consulted (discussions*, texts, urls, etc.) on this assignment. **Without this acknowledgement, your assignment will not be graded.** *Non-detailed oral discussion with others (inc. non-students) is permitted as long as any such discussion is summarized and acknowledged by all parties; the viewing or exchanging of any written work, even in rough or preliminary form, is expressly forbidden, as is any detailed discussion.
2. Recall: a k -partition of a non-empty n -set S is a k -set P whose elements are non-empty subsets of S , such that each element of S is in exactly one element of P . For example, $\{\{a, b, d\}, \{c\}, \{e\}\}$ is a 3-partition of $\{a, b, c, d, e\}$. $S(n, k)$, the number of k -partitions of an n -set, is given by $S(n-1, k-1) + kS(n-1, k)$.
 - (i) List all 2-partitions of $\{a, b, c, d, e\}$.
 - (ii) Explain why $\{\{\}, \{a, b\}\}$ is not a 2-partition of $\{a, b\}$.
 - (iii) Explain why $S(0, 0)$ is not defined.
 - (iv) Prove: for $n \geq 1$, $S(n, 1) = 1$.
 - (v) Prove: for $n \geq 2$, $S(n, 2) = 2^{n-1} - 1$.
 - (vi) Prove by induction: for $n \geq 2$, $S(n, 2) = (\sum_{m=1}^{n-1} S(m, 1)) + (\sum_{m=2}^{n-1} S(m, 2))$.
Hint: use (iv) and (v).
3. In each case, draw the “directed graph diagram” of the given relation: label points with elements of the underlying set X , and put an arrow from x to y if and only if xRy .
 - (i) $X = \{1, 2, 3, 4, 5\}$; $R = \{(2, 1), (3, 1), (4, 2), (2, 4), (3, 3)\}$
 - (ii) $X = \{1, \dots, 10\}$, and xRy if and only if $x = y \pmod{3}$.
4. For each of the following functions $f : \{1, 2, 3, 4\} \rightarrow \{?, <, >, +\}$ specified in one-line notation, (i) draw the directed graph diagram representing f , (ii) represent f as a relation by listing the set of ordered pairs, (iii) explain whether f is an injection, (iv) explain whether f is a surjection, (v) explain whether f is a bijection.
 - (a) $(>, >, +, +)$
 - (b) $(>, ?, +, <)$
5. (i) For each function from $\{1, 2, 3\}$ to $\{a, b, c\}$ with image size 2, give the directed graph diagram. (ii) Verify that the number of diagrams in (i) matches the formula given in the text.
6. Give (i) the number of onto functions from a 4-set to a 3-set, (ii) the number of 1-1 functions from a 3-set to a 4-set. Justify your answers.
7. Find the smallest t , such that each t -subset of $\{1, 2, \dots, 101\}$ contains two integers that are not relatively prime.