Asn. 3 CMPUT 272 posted Oct 21 due before 1230 Nov 1
in CMPUT 272 box opposite room CSC 145

1. Write a statement acknowledging **all** resources consulted (discussions*, texts, urls, etc.) on this assignment. **Without this acknowledgement, your assignment will not be graded.** *Non-detailed oral discussion with others (inc. non-students) is permitted as long as any such discussion is summarized and acknowledged by all parties; the viewing or exchanging of any written work, even in rough or preliminary form, is expressly forbidden, as is any detailed discussion.

2. (i) Prove/disprove: 5 is a Fermat prime. (ii) Prove/disprove: 7 is a Fermat prime. (iii) Can there exist a straightedge and compass construction of a regular 35-sided polygon? Justify carefully.

3. (i) Prove/disprove: for all integers n, if $n = 3(\text{mod}4)$ then $5n = 3(\text{mod}4)$. (ii) Prove/disprove: for all integers n, if $n = 3(\text{mod}4)$ then $\lfloor n/2 \rfloor = 1(\text{mod}2)$.

4. (i) Prove/disprove: $\forall x \in R$, $\forall n \in Z$, $\lceil n + x \rceil = n + \lceil x \rceil$. (ii) Compute $\lceil 1.3 + \lfloor 2.5 \rfloor \rceil$ and $\lfloor 1.3 + \lceil 2.5 \rceil \rfloor$. (iii) Prove/disprove: $\forall x \in R$, $\forall y \in R$, $\lceil x + \lfloor y \rfloor \rceil = \lfloor x + \lceil y \rceil \rfloor$.

5. (i) Find integers n, a, b such that $n = 24a + 15b$, $n|24$, $n|15$, and $n \neq \text{gcd}(15{,}24)$. (ii) Prove/disprove: for any integers x, y with x, y not both 0 and any positive integer n, if $n|x$ and $n|y$ and there are integers a, b such that $n = ax + by$, then $n = gcd(x, y$.

6. Recall: by the uniqueness of prime factorization, for every integer $n \geq 2$ there is an integer $t \geq 1$, a set of primes $\{p_1, \ldots p_t\}$, and a set of positive integers $\{e_1, \ldots, e_t\}$ such that $n = \prod_{j=1}^{t} p_j^{e_j}$. (i) Prove: $\forall n \in Z$, $5|n^3 \Rightarrow 5|n$. (ii) Use (i) to prove that $\sqrt[3]{5}$ is irrational.

7. (i) Using Euclid's algorithm, determine gcd(567,322). (ii) Using your work from (i), find integers a, b such that gcd(567,322) $= a \times 567 + b \times 322$. (iii) Without using prime factorization, determine lcm(567,322). (iv) Using prime factorization, determine gcd(567,322) and lcm(567,322).

8. Study §2 (Euclid's alg'm) from the handout (Variants/Invariants). Solve the problem in §2.5: show that the listed program is correct.