question 2 revised Oct 3

Asn. 2 CMPUT 272 posted Sept 27 due before 1230 Oct 11
in CMPUT 272 box opposite room CSC 145

1. Write a statement acknowledging **all** resources consulted (discussions*, texts, urls, etc.) on this assignment. **Without this acknowledgement, your assignment will not be graded.** *Non-detailed oral discussion with others (inc. non-students) is permitted as long as any such discussion is summarized and acknowledged by all parties; the viewing or exchanging of any written work, even in rough or preliminary form, is expressly forbidden, as is any detailed discussion.

2. A partially completed (correctly) sudoku puzzle has, for cells [4,5], [4,6], [8,5], [8,6], possible value sets {5,8,9}, {5,9}, {5,9}, {5,9} respectively. (i) If some solution has [4,5] = 5, is there necessarily a solution with [4,5] = 9? Justify carefully. (ii) If there is a unique solution, what are the possible values of [4,5]? Justify carefully.

3. A *Latin square* is an $n \times n$ array of n different symbols, each occurring exactly once in each row and exactly once in each column. (i) List all 2×2 Latin squares whose symbols are $1, 2$. (ii) Two Latin squares are *isotopic* if one can be obtained from the other by permuting rows and/or columns. Find the number of 4×4 pairwise nonisotopic Latin squares (Hint: you can assume that the symbols are $1, 2, 3, 4$; that the first row is (1,2,3,4); and that the first column is (1,2,3,4). Why?)

4. Read §1 (coffee can problem) of *Variants and Invariants*. In §1.5 (p7-8), answer parts 1,2,3 for variations B,C. Be brief in answering part 3.

5. Write the negation, converse, inverse, and contrapositive of the following. For each, give the truth value. Justify briefly.

$$\forall n \in \mathcal{N}, (\ (n \geq 3) \ \Rightarrow \sim (\exists a, b, c \in \mathcal{Z}, a^n + b^n = c^n)\ )$$

6. Let α be "For every positive integer x there is some non-negative integer y such that there is no prime z so that $x^2 + y^2 = z^3$." (i) Express α in symbolic form, without using any words. (ii) In words, give the negation of α.

7. Let the domain of x, y, z be $\mathcal{R}$. Negate and simplify the following.
   (a) $\forall x \forall y,\ (x \neq y) \Rightarrow (x - y)^2 > 0$.
   (b) $\forall x \forall z \exists y,\ (x^2 < z^2) \Rightarrow (\exists z, (x^2 < y^2 < z^2))$.

8. Let $y \in \mathcal{N}, y \geq 2$, and let D be the positive divisors of y. Prove that (i) the smallest element of D is 1, and (ii) the 2nd-smallest element of D is prime.

9. Let $p_1, p_2, \ldots, p_t$ be the t smallest prime numbers, and let $f(t) = 1 + p_1 \times p_2 \times \ldots \times p_t$. Prove or disprove: (i) p_1 does not divide $f(t)$ (ii) $f(t)$ is prime.

10. (i) Draw all possible ways four stones (2 black, 2 white) can be placed on a 2×2 Hex board. (ii) Assume two players play Hex on a 2×2 board, and that each plays randomly. What is the probability that the first player wins? Explain briefly.