

1. Acknowledge **all** resources consulted (discussions*, texts, urls, etc.) in working on this assignment. **Without this acknowledgement, your assignment will not be graded.** *Non-detailed oral discussion with others (inc. non-students) is permitted as long as any such discussion is summarized and acknowledged by all parties; the viewing or exchanging of any written work, even in rough or preliminary form, is expressly forbidden, as is any detailed discussion.

2. Prove by induction: for all integers $n \geq 1$, $\sum_{t=1}^n (2t+1) = (n+1)^2 - 1$.

3. (i) Using a truth table, prove/disprove:

$$(\sim x) \vee z = \sim (x \vee y) \vee \sim (y \vee \sim z) \vee \sim (x \vee \sim y) \vee \sim (\sim y \vee \sim z) .$$

(ii) Prove/disprove the above using boolean algebra.

4. Express the following boolean function $f(p, q, r)$ (i) in disjunctive normal form (ii) in conjunctive normal form (iii) using only boolean operators \sim, \vee (iv) using only boolean operators \sim, \wedge . Show your work; simplify your answers.

| p | q | r | f |
|-----|-----|-----|-----|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |

5. Show your work.

(i) Convert $AF013_{16}$ to octal and binary. (ii) Convert 807263_9 to base 3. (iii) Convert 10210122_3 to base 9. (iv) Convert 12345_6 to base 11 (use A as the symbol for ten).

6. Show your work. (i) Using base 2 arithmetic, compute $69 - 55$. (ii) Compute $69 + -55$ using-8 bit two's-complement registers; check for overflow.

7. In English, for each statement, give the converse, inverse, and contrapositive. Make your answers easy to read.

(a) If Squarepants is a liar, then I'm wrong.

(b) If you are Kim's sibling, then I'm your ancestor and you are Mary Boole's relative.

8. Rewrite the following using only (i) \sim, \vee (ii) \sim, \wedge . Simplify.

(a) $(\sim p \vee q) \Rightarrow (r \wedge \sim q)$

(b) $((p \Rightarrow q) \Rightarrow r) \Leftrightarrow ((p \vee q) \Rightarrow r) \vee \sim p$

9. (i) Complete only as much of the following sudoku puzzle as you can **without making a guess or choice at any stage**. List your deductions in order, and for the first 5 cells filled, explain your deduction. E.g., your answer might start like this:
1. [9,7] ← 3 by crosshatching.
 2. ...

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 5 | | | 7 | | | 8 | 3 |
| | 9 | 8 | | | 4 | | | |
| 6 | 3 | | 1 | | | | | |
| 7 | | 6 | | | 1 | | | |
| | | 4 | | | | 9 | | |
| | | | 7 | | | 1 | | 2 |
| | | | | | 3 | | 9 | 7 |
| | | | 5 | | | 6 | 1 | |
| 8 | 6 | | | 1 | | | 2 | |

- (ii) Finish solving the puzzle.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 5 | | | 7 | | | 8 | 3 |
| | 9 | 8 | | | 4 | | | |
| 6 | 3 | | 1 | | | | | |
| 7 | | 6 | | | 1 | | | |
| | | 4 | | | | 9 | | |
| | | | 7 | | | 1 | | 2 |
| | | | | | 3 | | 9 | 7 |
| | | | 5 | | | 6 | 1 | |
| 8 | 6 | | | 1 | | | 2 | |

- (iii) How many solutions does the puzzle have? Justify your answer.