

time: 50 minutes  
pen/pencil only

CS 204 Midterm  
write all answers in the space provided

Friday March 1 2002  
marks: 17

1. (1 + 1 marks) (a) Replace each of the following functions  $f(n)$  with the simplest function in  $\Theta(f(n))$ .  
(b) Rank the functions by increasing order of complexity (1 for smallest); give functions in the same  $\Theta$  class the same rank.

$$4^n + n^4 \lg n \qquad 99n^2 \lg n \qquad \frac{n^3}{7 \lg^7 n} \qquad 7^{\lg n} \qquad (n/2) \lg(n/2)$$

2. (1 + 1 marks) For the following recurrence relations, as accurately and simply as you can, give the asymptotic complexity of  $T(n)$ .

(a)  $T(n) = 9T(n/3) + 13n^2$  for  $n \geq 2$ ,  $T(1) = 1$ .

(b)  $T(n) = 13T(n/2) + 7n^2$  for  $n \geq 2$ ,  $T(1) = 1$ .

3. (4 marks) Trace quicksort on the array [3 8 6 9 2 1 7 4 5] by drawing the recursion tree. Label each node with the subarray to be sorted. Also, for each node, give the number of key comparisons (KC) and interchanges (I) performed by partition.

4. (0.5 + 1 + 2 marks) (a) For a rooted binary tree (each node has at most 2 children),
- (i) what is the maximum number of nodes if the depth is  $d$ ?
  - (ii) what is the minimum depth if the number of nodes is  $n$ ?
- (b) Using the above, prove a run time lower bound for some sorting algorithms.

5. (2.5 + 3 marks)

```
for j <- 2 to n do
  k <- j
  while k > 1 and A[k] > A[k/2] do
    interchange A[k] <-> A[k/2]
    k <- k/2
```

- (a) For the above pseudocode, give the worst case run time. Justify.

- (b) Prove (using a loop invariant) or disprove (with a counterexample): upon completion,  $A[1]$  is a largest element of  $A[1 \dots n]$ .