*LAST NAME:*____________ *Initial:*____________ *ID number:*____________ *Section:*____________

CS 204 Midterm Wedneday October 3 2001

Write all answers in the space provided. Time: 50 minutes. Total Marks: 20

1. $(1+2+2=5$ marks)

(a) Give a precise mathematical definition of $O(n \lg n)$.

(b) For each of the following functions, give (i) the simplest Θ expression (ii) its rank by decreasing order of complexity (the largest gets rank 1; functions with the same order get the same rank).

	$99n^2 + \frac{n^9}{9\lg n} + 99n^7$	$8^{\lg n}$	$(1+n\lg n)^2$	$\sum_{j=1}^{n}(n+\lg j)$
(i)				
(ii)				

2. $(1+2+2=5$ marks)

```
proc blah(n)   (* a/b is integer division, round down*)
  if n < 4 then
    return 1*2 + 3
  else
    return blah(n/4) + 2*blah(n/3) + blah(n/2)
```

(a) Give a recurrence relation for $A(n)$, the number of `+ * /` operations performed by `blah(n)`.

(b) Draw the recursion tree for `blah(9)` and determine $A(9)$.

(c) Omitting the base cases, give a recurrence relation for $A(n)$ in which $A(n/2)$ does not appear. Hint: substitute once.

3. (3 marks) For the following recurrence relations, use the master theorem to give the simplest Θ complexity of $R(n)$, or explain why the theorem does not apply.

   (a) $R(n) = 17R(n/2) + n^4 + 3n^2 \lg n$ for $n \geq 2$, $R(1) = 1$.

   (b) $R(n) = 125R(n/5) + 7n^3$ for $n \geq 2$, $R(1) = 1$.

4. (3 marks) Suppose that insertion sort is implemented with an array, with binary search to find insert locations. Using Θ notation describe, for n keys, the worst case (a) number of key comparisons (b) running time. Justify briefly.

5. (4 marks) State and prove an invariant which holds each time execution reaches the start of line 2, and which could be used to show that upon termination, $A[1]$ is the maximum key in $A[1, ..., n]$ (you do **not** have to show that $A[1]$ has this property).

```
1  for j <- n-1 downto 1 do
2   if A[j] < A[j+1] then
3     interchange A[j] <-> A[j+1]
```