# Lecture 34: Wednesday April 9, 2003

## today

- last lecture
- NP completeness
- quick review

## announcements

- final exam

## recall: how to show that a problem B is in NP-c

- use polynomial reduction (transformation)
  - show that B is in NP — usually easy
  - find an 'known' problem A (known to be in NP-c) — e.g. SAT
  - poly'ly reduce A to B
- consequence
  - any problem in NP can be reduced into A
  - A can be reduced into B ...so
  - any problem in NP can be reduced into B ...so
  - B is in NP-c
  - picture:

## examples of polynomial reductions

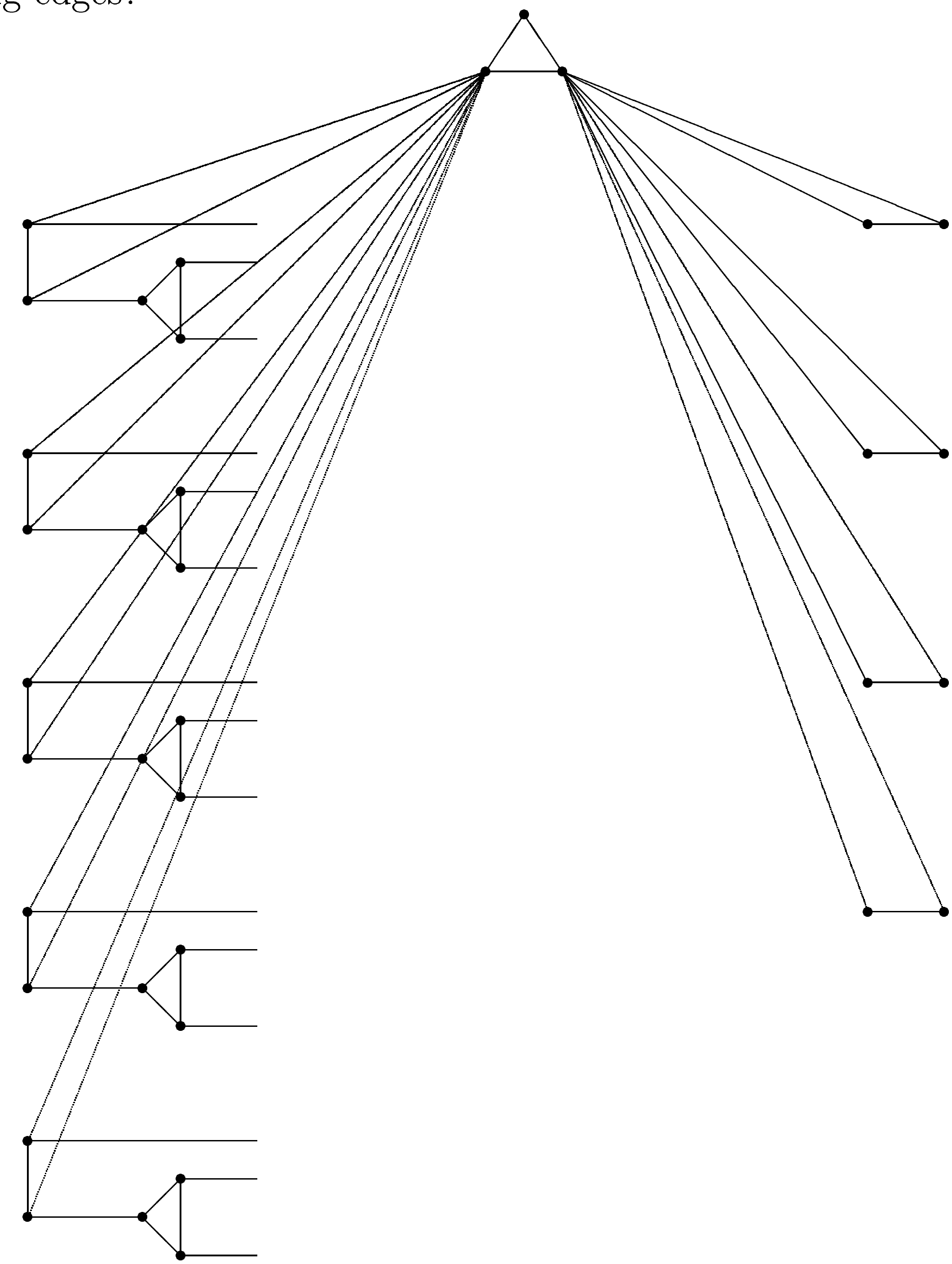
- CNF-SAT reduces to 3-CNF-SAT — done
- 3-CNF-SAT reduces to 3-colouring — done
- 3-colouring reduces to 4-colouring — done
- lots more in [CLRS Ch. 34]

## recall reduction: 3-CNF-SAT to 3-colouring

- transformation input: 3-CNF-SAT formula $f = C_1 \wedge C_2 \wedge \ldots \wedge C_m$ where variables = $x_1, \ldots, x_n$ and each $C_k = (C_{k,1} \vee C_{k,2} \vee C_{k,3})$
- transformation output: graph $G_f$, 3-colourable iff $f$ sat'ble
- how to construct $G_f$ from $f$:
  - vertices
    - $t, f, g$
    - $x_j$ and $\overline{x}_j$ — for $j = 1$ to $n$
    - $a_k, b_k, c_k, d_k, e_k$ — for $k = 1$ to $m$
  - edges
    - $(tf)$ $(tg)$ $(fg)$
    - $(gx_j)$ and $(g\overline{x}_j)$ — for $j = 1$ to $n$
    - $(x_j\overline{x}_j)$ — for $j = 1$ to $n$
    - $(a_kb_k)$ $(a_kc_k)$ $(b_kc_k)$ $(c_kd_k)$ $(d_ke_k)$ — for $k = 1$ to $m$
    - $(td_k)$ $(te_k)$ $(e_kC_{k,1})$ $(b_kC_{k,2})$ $(a_kC_{k,3})$ — for $k = 1$ to $m$

$f = (x_1\overline{x}_2x_3) \wedge (x_1x_2\overline{x}_4) \wedge (\overline{x}_1\overline{x}_3x_4) \wedge\ (x_2x_3\overline{x}_4) \wedge (\overline{x}_2\overline{x}_3x_4)$

missing edges?



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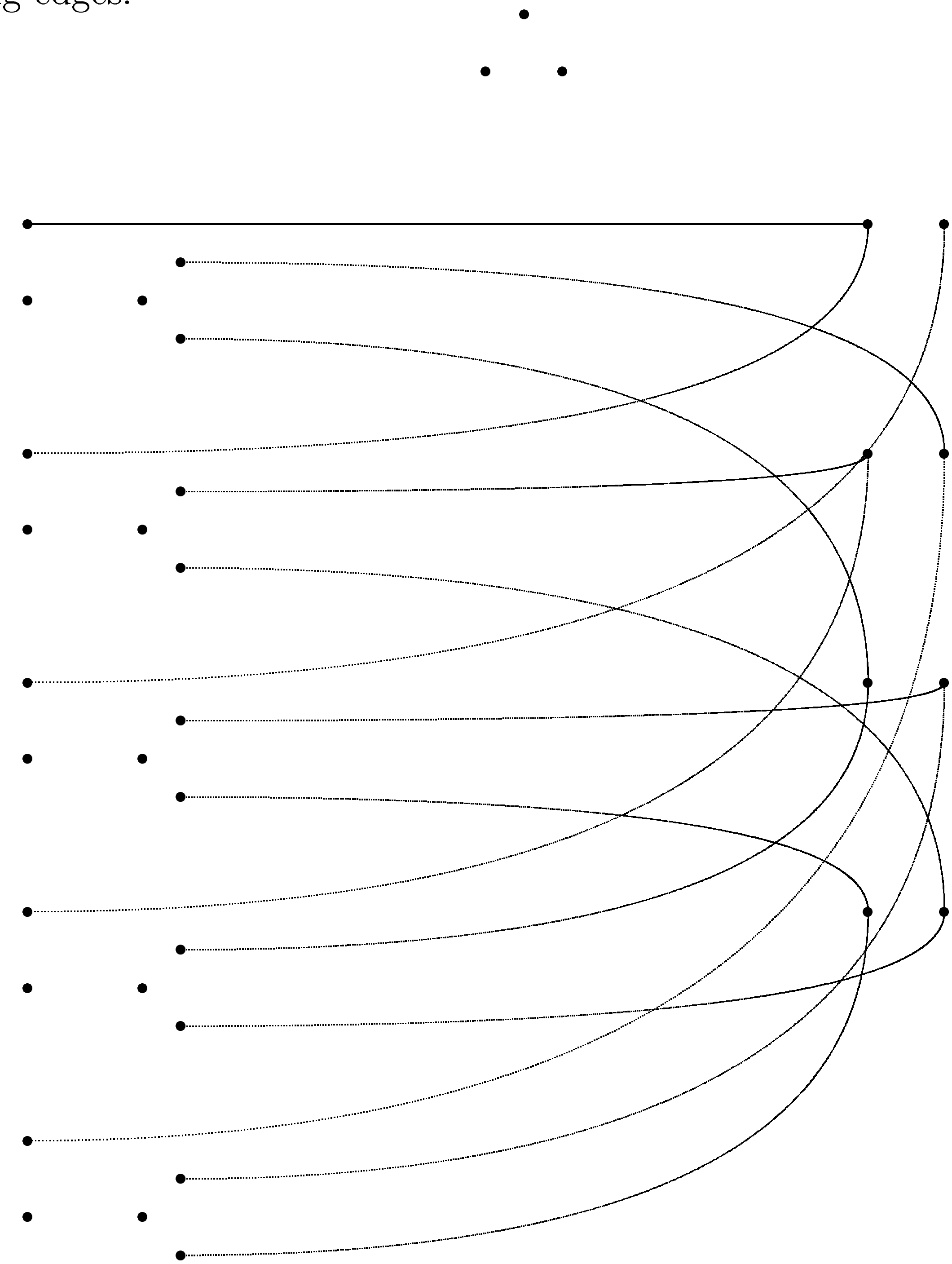
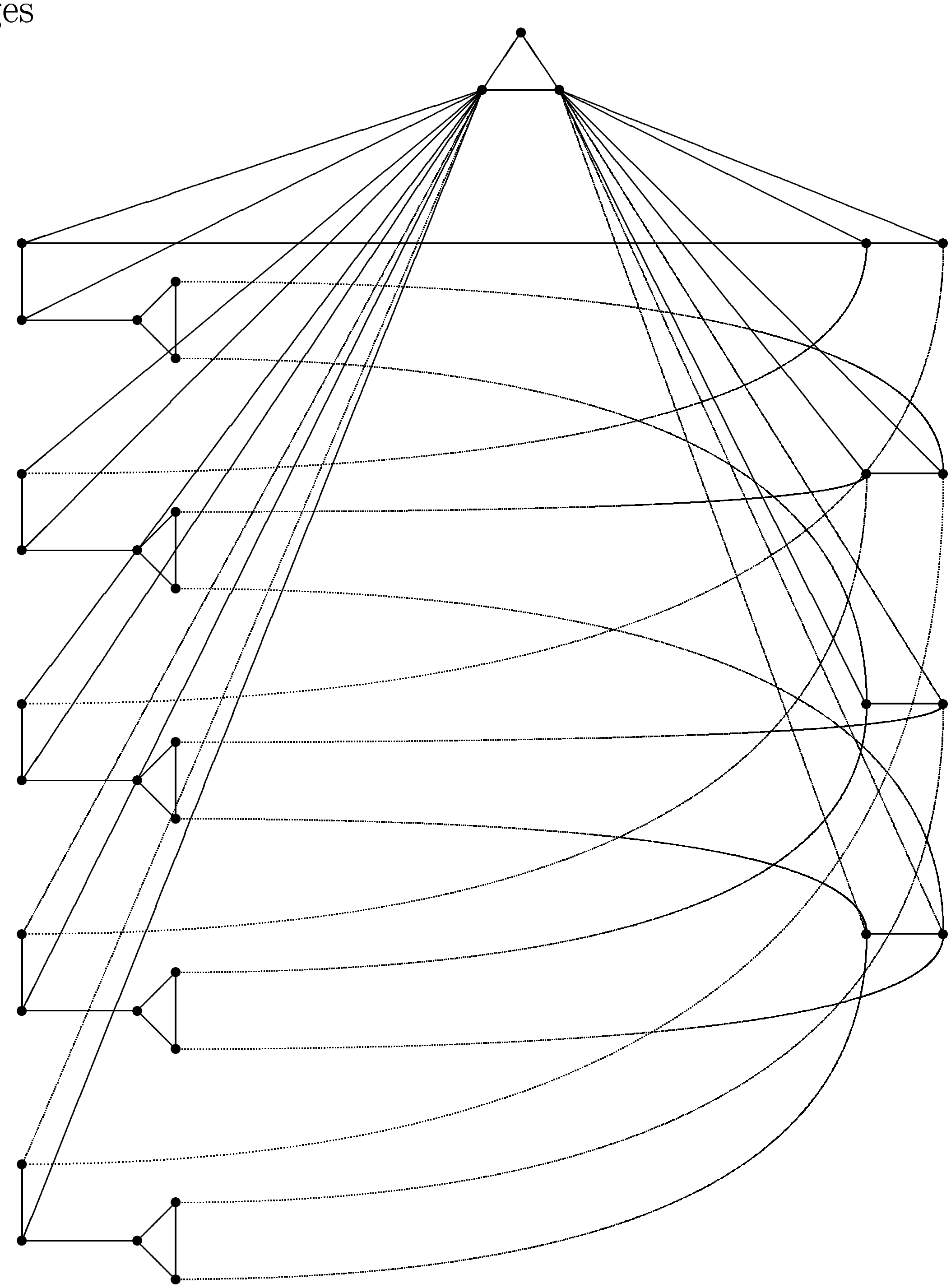
missing edges!



$f = (x_1\overline{x}_2x_3) \wedge (x_1x_2\overline{x}_4) \wedge (\overline{x}_1\overline{x}_3x_4) \wedge\ (x_2x_3\overline{x}_4) \wedge (\overline{x}_2\overline{x}_3x_4)$

all edges



**consequence**

- 3-CNF-SAT is known to be in NP-c ...
- 3-colouring is in NP ... (exercise)
- 3-CNF-SAT is polynomial-time-reducible to 3-colouring ...
- ...so ...3-colouring is in NP-c

## final exam

- cumulative, mostly post-midterm material
- some topics covered . . .
- graph algorithms
  - DFS, BFS, their properties
  - BFS and distance
  - DFS and biconnected components
  - MST (Prim; Kruskal)
  - single source shortest path (non-negative weights) Dijkstra
- divide and conquer matrix multiplication (Strassen)
- NP-completeness
  - abstract problem, decision problem
  - P, NP, co-NP, NP-c
  - SAT, CNF-SAT, k-CNF-SAT
  - Cook's theorem
  - reductions
    - CNF-SAT to 3-CNF-SAT
    - 3-CNF-SAT to 3-colouring
    - 3-colouring to 4-colouring
    - more in the text

**so long, and good luck** ☹ ☺ ☹ ☺ ☹ ☺ . . .