

Lecture 26: Monday March 17, 2003

today

- DFS application: biconnected components

announcements

recall connected components

- *connected graph* for any vertices x, y , some $x - y$ path
- *connected component (aka component)*
maximal connected subgraph
- example

biconnected components

- *cut vertex* removal increases number of components
corollary: for some other x, y , on every x - y -path
- *biconnected graph* connected and no cut vertex
- *biconnected component (aka bicomponent)*
maximal biconnected subgraph
- example

rooted tree def'ns

- *rooted tree* tree with one root vertex
- in a rooted tree,
 - *ancestor* of v any vertex on v -to-root path
 - *proper ancestor* ancestor other than vertex itself
 - z *descendant* of v iff v ancestor of z
 - *proper descendant* descendant other than vertex itself
 - *parent* of v neighbour of v on v -to-root path
 - z *child* of v iff v parent of z

warmup: how to find components of a graph?

- hint: modify dfs
- for each vertex, store `cmptVal`
- initialize `cmptNum` to 0
- in outer `for` loop
 - when WHITE vertex encountered increment `cmptNum`
- in `DFS-visit(v)`
 - when `colour[v] <- GRAY` `cmptVal[v] <- cmptNum`
- run time?
 - adj. list representation $\Theta(n + m)$
 - adj. matrix representation $\Theta(n^2)$
- example

- exercise: find components by modifying bfs

how to find bicomponents of a graph?

- easier problem: how to find cut vertices?
- simple alg'm?

```
alg'm simpleCutVertex(G)
1 numC <- numberOfComponents(G)
2 for each vertex v do
3   G' <- G-v
4   nC <- numberOfComponents(G')
5   if nC > numC then
6     report 'v is cut vertex of G'
```

- correctness?

from the def'n ☺

- run time?

– adj. list

$\Theta(n^2 + nm)$ (why?)

– adj. list

$\Theta(n^3)$ (why?)

- can do better



finding bicomponents based on dfs tree

- easier problem: using dfs tree, find cut vertices
- given dfs tree of graph
 - what vertices **cannot** be cut vertices of graph? leaves (why?)
 - when is root cut vertex of graph? iff has ≥ 2 children (why?)
- observe: v cut vertex of graph iff, w.r.t. dfs tree

root: more than one subtree

not root: child subtree has no back edge to proper v -ancestor

- above observations are basis of bicomponent alg'm