Lecture 25: Friday March 14, 2003

today

- graph traversal (finish)
  - BFS and DFS trees
  - classifying graph edges with BFS/DFS

next

- DFS application: biconnected components

announcements

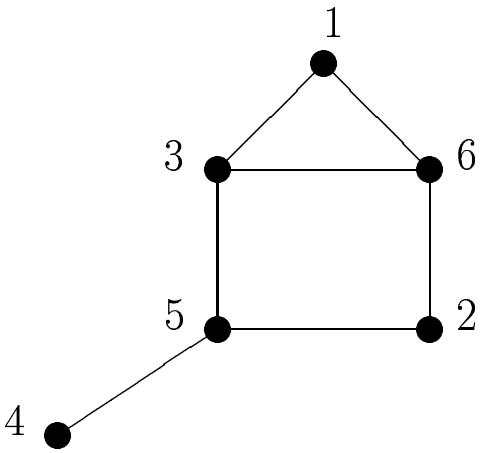
depth first search: recursive traversal

```
DFS(G)                    { G=(V,E)
  for each v in V do
    color[v] <- WHITE     { unknown
    p[v]     <- NIL       { predecessor
  time <- 0
  for each v in V do
    if color[v]=WHITE then DFS-visit(v)

DFS-visit(u)
  color[u] <- GRAY                { discover u
  dtime[u] <- time  <- 1+time
  for each v in Adj[u] do         { nbrs of u
    if color[v]= WHITE then
      p[v] <- u
      DFS-visit(u)
  color[u] <- BLACK               { finished u
  ftime[u] <- time  <- 1+time
```

WARNING: DFS(G) versus BFS(G,s)

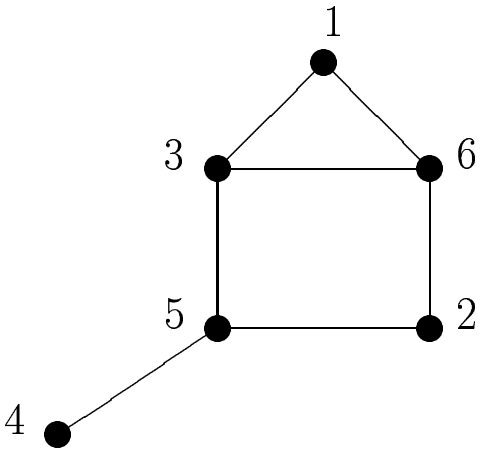
- DFS(G): traverses all components of the graph
- DFS-visit(u): never leaves the component containing u
- BFS(G,s): never leaves the component containing s
- exercise: write BFS(G) which traverses all components




```
1:  3 6
2:  5 6
3:  1 5 6
4:  5
5:  2 3 4
6:  1 2 3
```

	1	2	3	4	5	6
1			*			*
2					*	*
3	*				*	*
4					*	
5		*	*	*		
6	*	*	*			

DFS(G)




```
1:  6 3
2:  6 5
3:  6 5 1
4:  5
5:  4 3 2
6:  3 2 1
```

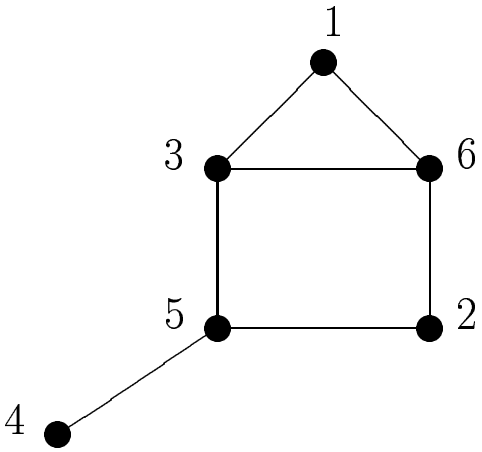
DFS(G)

DFS analysis

- similar to BFS
- $n = |V|$ $m = |E|$
- recall handshaking lemma: $\sum_{v \in V} deg(v) = 2|E|$
- DFS analysis:
  - adjacency matrix representation
    - space $\Theta(n^2)$
    - time $\Theta(n + \sum_v n = n + n^2)$ $\Theta(n^2)$
  - adjacency list representation
    - space $\Theta(n + \sum_v d(v) = n + 2m)$ $\Theta(n + m)$
    - time $\Theta(n + \sum_v d(v) = n + 2m)$ $\Theta(n + m)$

classifying graph edges with BFS/DFS

- given a BFS/DFS forest, where are the possible other edges?
- during graph traversal, all edges/vertices examined
- traversal forest:
  - tree root: starting vertex for that component
  - tree edge (parent p, child c): c discovered while processing p
- during graph traversal, each edge encountered twice (from each end)
- wrt traversal tree, categorize graph edges by first encounter
  - tree edges
  - graph edges which are not tree edges are called:
    - back edges — to ancestor
    - forward edges — to descendant
    - cross edges — to non-ancestor, non-descendant




```
1:  3 6
2:  5 6
3:  1 5 6
4:  5
5:  2 3 4
6:  1 2 3
```

BFS(G,6)

DFS-visit(6)

properties of BFS, DFS

- classifying graph edges with BFS
  - each graph edge: node levels differ by ≤ 1 why?
  - so no back edges
  - so no forward edges
  - cross edges
    - level edges within same level
    - downleft edges child of previous node at same level
- classifying graph edges with DFS
  - every non-tree edge is back edge why?
  - so no forward edges
  - so no cross edges
- BFS vertex order level-order of each tree of BFS forest
- DFS vertex order pre-order of each tree of DFS forest

rooted tree traversals: level/pre/in/post-order

- each describes an order for visiting all nodes
- in rooted tree, children of a node:
  - arbitrary number, ordered first to last
  - when drawing: first $\leftrightarrow$ leftmost, last $\leftrightarrow$ rightmost

```
level-order(T)
  visit(root of T)
  visit nodes at level 1 (children of root) left to right
  visit nodes at level 2 (grandchildren " ) left to right
  ...

pre-order(T)
  visit(root of T)
  pre-order(first subtree)
  ...
  pre-order(last subtree)

in-order(T)                { only defined for binary trees
  in-order(left subtree)
  visit(root of T)
  in-order(right subtree)

post-order(T)
  post-order(first subtree)
  ...
  post-order(last subtree)
  visit(root of T)
```

DFS discovery/finish times

- useful in proving correctness for DFS algorithms

```
DFS-visit(u)
  color[u] <- GRAY              { discover u
  dtime[u] <- time  <- 1+time
  for each v in Adj[u] do       { nbrs of u
    if color[v]= WHITE then
      p[v] <- u
      DFS-visit(u)
  color[u] <- BLACK             { finished u
  ftime[u] <- time  <- 1+time
```

- DFS parenthesis theorem: for x,y with disc[x] $<$ disc[y]
  - disc[x] $<$ finish[x] $<$ disc[y] $<$ finish[y],
    and neither is descendant of other in DFS forest
  - disc[x] $<$ disc[y] $<$ finish[y] $<$ finish[x]
    and y is descendant of x in DFS forest