Lecture 14: Wednesday February 5, 2003

today

- QS analysis and recursion trees
- a bad case for QS (actually WC)
- a good case for QS (actually BC)

QS analysis and recursion trees

- QS run time proportional to number key comparisons why?
- QS recursion tree makes counting KC easy, because
  to count KC, only need know (at each call) rank of split key

```
a 16 key example: a 16 key QS recursion tree, showing rank of
           - split key (if partition called)
           - single key (if sublist has size 1)

                             [1,16]
                               06

              [1,5]                           [7,16]
               04                               09

       [1,3]        [5,5]           [7,8]                 [10,16]
        02           05              07                     16

  [1,1]     [3,3]               [*-*]   [8,8]        [10,15]    [*-*]
   01        03                          08             15

                                              [10,12]       [14,15]
                                                10             14

                                             [*-*] [11,12] [*-*] [15,15]
                                                     12             15

                                              [11,11] [*-*]
                                                11
```

- observe: in resulting tree, for each node,
  all keys(left subtree) $\leq$ node's key $\leq$ all keys(right subtree)
- 1-1 correspondence: quicksort recursion tree $\leftrightarrow$ binary search tree

QS run time: a bad case

```
01
     02
         03
              04
                  05
                      06
                          . . .
```

- number of KC in above case?
- $W(n) = \begin{cases} 0 & \text{if } n \leq 1 \\ n-1+W(0)+W(n-1) & \text{if } n \geq 2 \end{cases}$
- $W(n) \;=\; \sum_{j=n-1}^{1} j \;=\; n(n-1)/2$
- so QS WC KC $\geq n(n-1)/2$

QS run time: worst case

- previous example is a worst case, because
- QS WC KC $\leq n(n-1)/2$:
  - each KC is between splitter/other
  - after partition, splitter never compared again
  - so each pair of elements compared ≤ 1 time
  - $\binom{n}{2} = n(n-1)/2$
- conclusion: QS WC
  - KC $= n(n-1)/2$
  - so run time in $\Theta(n^2)$

QS run time: a good case

08

04 12

02 06 10 14

01 03 05 07 09 11 13 15

- number of KC in above case?
- $B(1) = 0$
- $B(n) = n - 1 + B(\lfloor n - 1/2 \lfloor) + B(\lceil n - 1/2 \rceil)$ why?
- $B(n) \approx n + 2B(n/2)$
- can show $B(n) \in \Theta(n \log n)$ e.g. use master theorem
- conclusion: QS BC KC in $O(n \log n)$
- is QS BC KC in $\Theta(n \log n)$?
- yes: can show (e.g. by induction) $B(n)$ is best case number of KC, so
- QS best case run time in $\Theta(n \lg n)$