Lecture 36: Wednesday December 5, 2001

today

- last lecture ☺
- NP completeness
- quick review

announcements

- final exam
  - section A1/EA1 — Tuesday Dec 11 9am-noon CSC B2
  - section A2/EA2 — Tuesday Dec 11 9am-noon MEC 23
  - section A3/EA3 — Tuesday Dec 18 2pm-5pm CSC B2

recall: how to show that a problem B is in NP-c

- use polynomial reduction (transformation)
  - show that B is in NP — usually easy
  - find an ‘known’ problem A (known to be in NP-c) — e.g. SAT
  - poly’ly reduce A to B
- consequence
  - any problem in NP can be reduced into A
  - A can be reduced into B …so
  - any problem in NP can be reduced into B …so
  - B is in NP-c
  - picture:

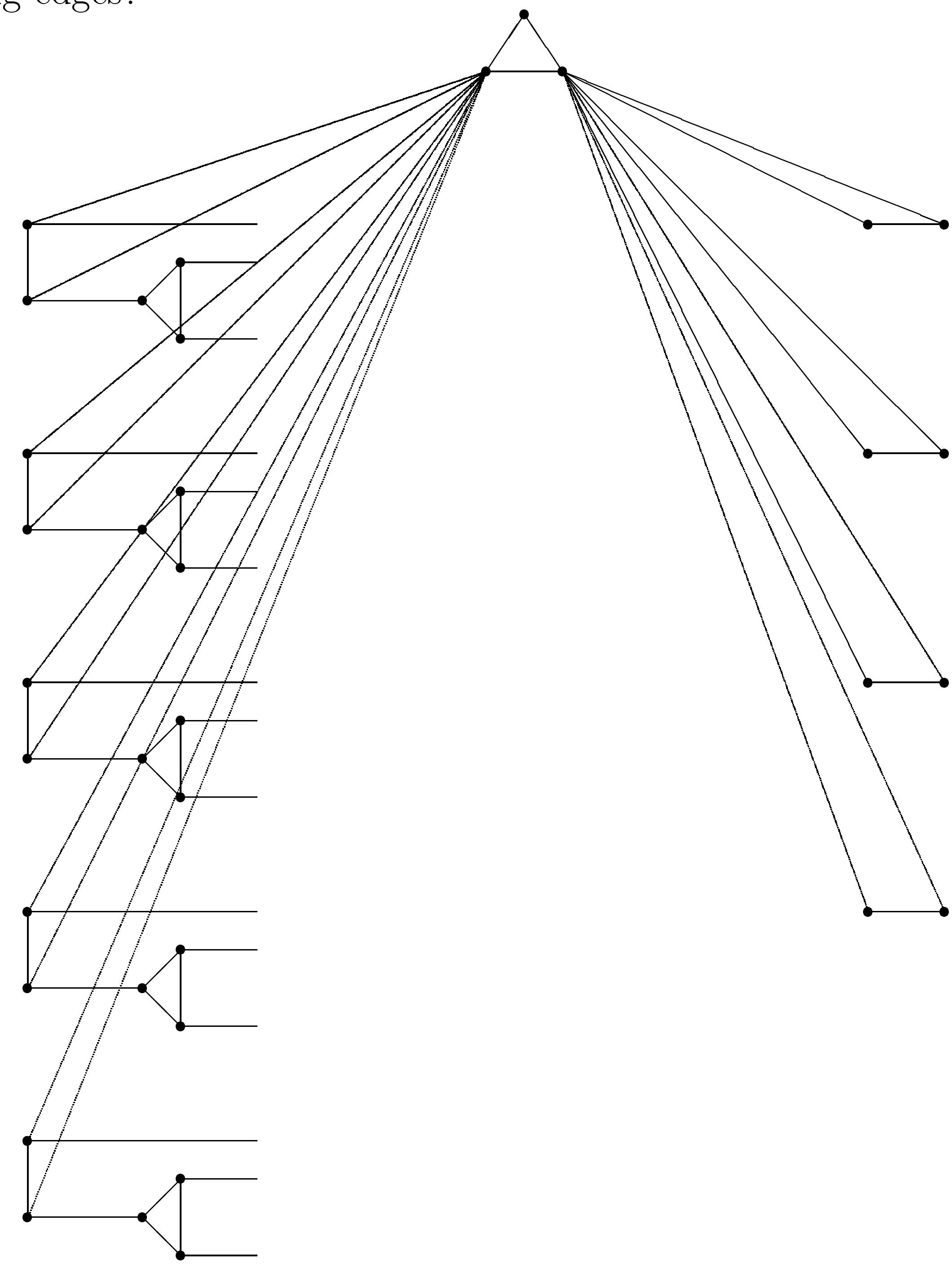
examples of polynomial reductions

- CNF-SAT reduces to 3-CNF-SAT done
- 3-CNF-SAT reduces to 3-colouring done
- 3-colouring reduces to 4-colouring done
- lots more in [CLRS Ch. 34]

recall reduction: 3-CNF-SAT to 3-colouring

- transformation input: 3-CNF-SAT formula $f = C_1 \wedge C_2 \wedge \ldots \wedge C_m$ where variables $= x_1, \ldots, x_n$ and each $C_k = (C_{k,1} \vee C_{k,2} \vee C_{k,3})$
- transformation output: graph G_f, 3-colourable iff f sat'ble
- how to construct G_f from f:
  - vertices
    * t, f, g
    * x_j and $\overline{x}_j$ — for $j = 1$ to n
    * a_k, b_k, c_k, d_k, e_k — for $k = 1$ to m
  - edges
    * (tf) (tg) (fg)
    * (gx_j) and $(g\overline{x}_j)$ — for $j = 1$ to n
    * $(x_j\overline{x}_j)$ — for $j = 1$ to n
    * (a_kb_k) (a_kc_k) (b_kc_k) (c_kd_k) (d_ke_k) — for $k = 1$ to m
    * (td_k) (te_k) $(e_kC_{k,1})$ $(b_kC_{k,2})$ $(a_kC_{k,3})$ — for $k = 1$ to m

$f = (x_1\overline{x}_2x_3) \wedge (x_1x_2\overline{x}_4) \wedge (\overline{x}_1\overline{x}_3x_4) \wedge\ (x_2x_3\overline{x}_4) \wedge (\overline{x}_2\overline{x}_3x_4)$
missing edges?



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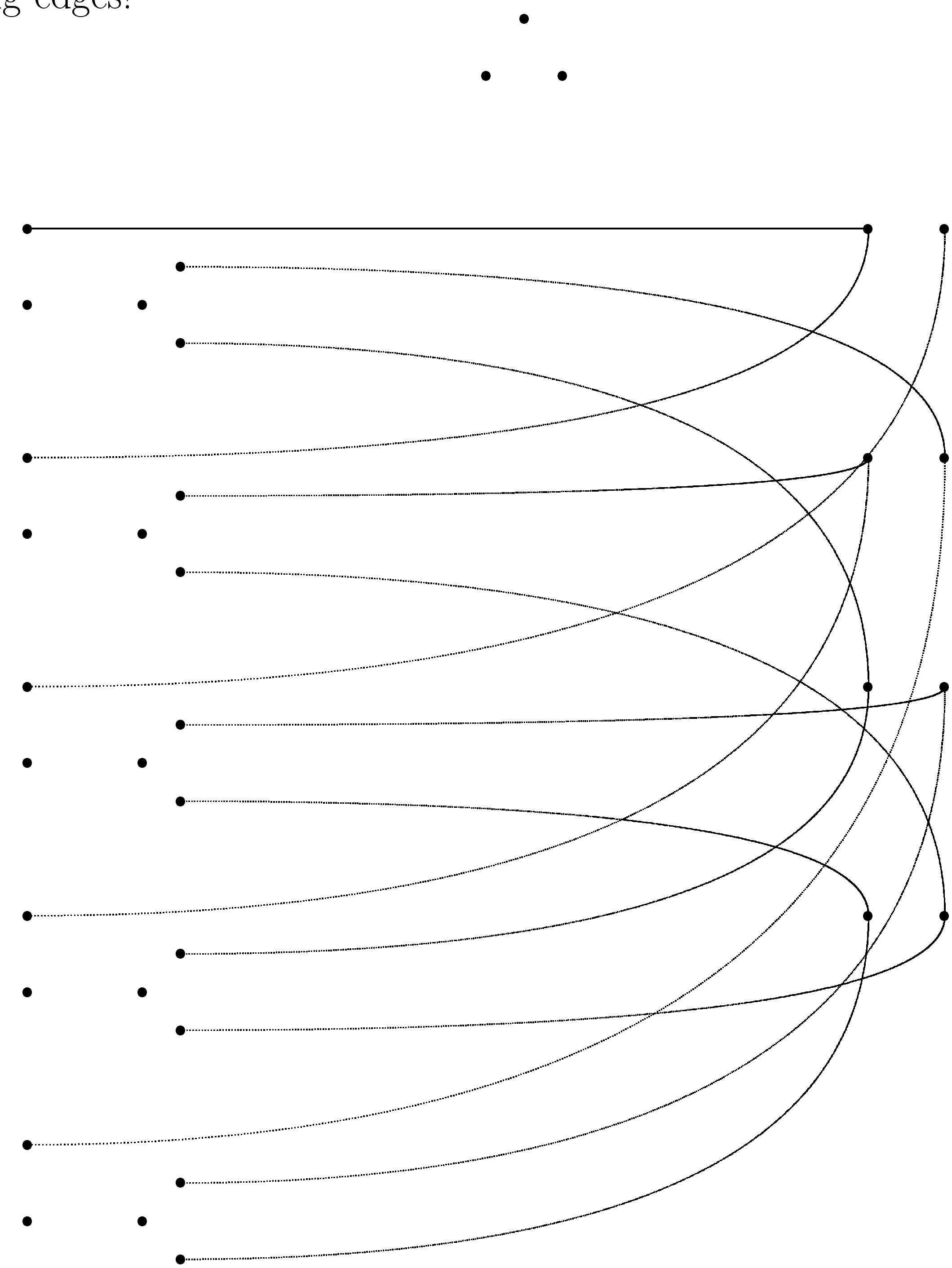
missing edges!



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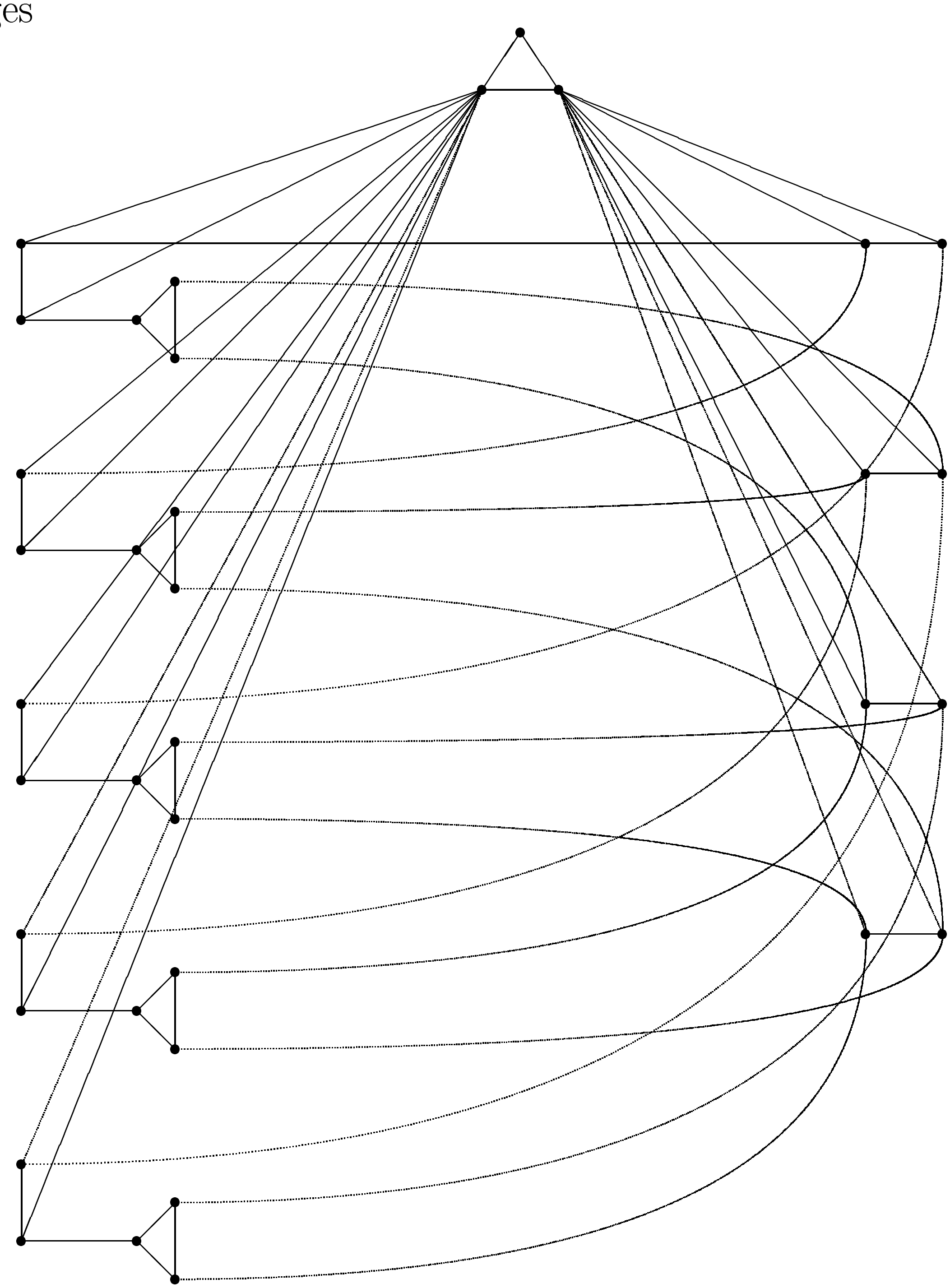
all edges



consequence

- 3-CNF-SAT is known to be in NP-c ...
- 3-colouring is in NP ... (exercise)
- 3-CNF-SAT is polynomial-time-reducible to 3-colouring ...
- ... so ... 3-colouring is in NP-c

final exam

- cumulative, but mostly since Midterm II
- some topics covered ...
- graph algorithms
  - DFS, BFS, their properties
  - BFS and distance
  - DFS and biconnected components
  - MST (Prim; Kruskal)
  - single source shortest path (non-negative weights) Dijkstra
- divide and conquer matrix multiplication (Strassen)
- NP-completeness
  - abstract problem, decision problem
  - P, NP, co-NP, NP-c
  - SAT, CNF-SAT, k-CNF-SAT
  - Cook's theorem
  - reductions
    - $*$ CNF-SAT to 3-CNF-SAT
    - $*$ 3-CNF-SAT to 3-colouring
    - $*$ 3-colouring to 4-colouring
    - $*$ more in the text

so long, and good luck ☹ ☺ ☹ ☺ ☹ ☺ ...