recall $3^{\lg n} = n \lg 3$

Proof: $x = 2^{\lg x}$ for $x > 0$. Let $x = 3^{\lg n}$. Then

$3^{\lg n} = 2^{\lg(3^{\lg n})} = 2^{(\lg n)\lg 3} = (2^{\lg n})^{\lg 3} = n^{\lg 3}$.

solving T(n) = 3T(n/2) + 5n

Define $T(1) = 1$ and $T(n) = 3T(n/2) + 5n$ for $n \geq 2$.

Assume $n = 2^k$ for some positive integer k.

$$
\begin{aligned}
T(n) &= 3T(n/2) + 5n \\
&= 3\,[\,3T(\frac{n/2}{2}) + 5(n/2)\,] + 5n \\
&= 3 \cdot 3T(n/4) + 3 \cdot 5(n/2) + 5n \\
&= 3 \cdot 3\,[\,3T(\frac{n/4}{2}) + 5(n/4)\,] + 3 \cdot 5(n/2) + 5n \\
&= 3^3T(n/8) + 3^2 \cdot 5(n/4) \;+\; 3^1 \cdot 5(n/2) \;+\; 5n \\
&= 3^3T(n/2^3) + (3/2)^2 \cdot 5n \;+\; (3/2)^1 \cdot 5n \;+\; (3/2)^0 \cdot 5n \\
&= 3^3\,[\,T(\frac{n/2^3}{2}) \;+\; 5(n/2^3)\,] \;+ (3/2)^2 \cdot 5n \;+\; (3/2)^1 \cdot 5n \;+\; (3/2)^0 \cdot 5n \\
&= 3^4T(n/2^4) +\; 5n[(3/2)^3 \;+\; (3/2)^2 \;+\; (3/2)^1 \;+\; (3/2)^0] \\
&= 3^5T(n/2^5) +\; 5n[(3/2)^4 \;+\; (3/2)^3 \;+\; (3/2)^2 \;+\; (3/2)^1 \;+\; (3/2)^0] \\
&= 3^kT(n/2^k) +\; 5n\sum_{j=0}^{k-1}(3/2)^j \\
&= 3^kT(1) +\; 5n\frac{(3/2)^k - 1}{(3/2) - 1} \\
&= 3^k +\; 10n\,((3/2)^k \;- 1\,) \\
&= 3^k +\; 10 \cdot 3^k(n/2^k) \;- 10n \\
&= 3^k +\; 10 \cdot 3^k \;- 10n \\
&\in \Theta(3^k) \;=\; \Theta(3^{\lg n}) = \;\Theta(n^{\lg 3}) \;=\; \Theta(n^{1.58\ldots})
\end{aligned}
$$